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# Dynamics of Oxygen-Phytoplankton Interactions: Understanding Global Warming through the Allee Effect

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**Abstract**

In this presentation, we investigate the dynamic behavior of an oxygen-phytoplankton interaction model [1] incorporating a strong Allee effect. To characterize how the system evolves, we employ both the continuous-time differential equation framework and its discrete-time counterpart [2]. Initially, the local stability conditions of the biologically relevant equilibrium points–extinction, Allee threshold, and positive coexistence–are determined for both systems. The prerequisites for the presence of specific bifurcations are next studied. We demonstrate that the continuous system undergoes a transcritical bifurcation driven by the Allee threshold, acting as a tipping point between survival and collapse. Conversely, the discrete-time system exhibits flip bifurcations leading to chaotic dynamics as the time step size varies [3]. We have observed that the Allee equilibrium functions as a critical separatrix; trajectories below this threshold collapse to extinction, while those above may converge to coexistence. To rigorously quantify the vulnerability of the system beyond standard stability analysis, we utilize the resilience index to estimate the probability of persistence based on the basin of attraction, and the first passage time function to detect critical slowing down near these tipping points. Our numerical simulations reveal a "ghost of attractor" phenomenon [4], indicating that the system may exhibit low resilience by lingering in a vulnerable depletion zone even before a total collapse. Ultimately, this study serves as a theoretical foundation for understanding the ecological consequences of global warming, demonstrating how environmental stress can push seemingly stable ecosystems across critical thresholds toward irreversible collapse.

**Keywords:** Allee effect, global warming, stability analysis, tipping point, bifurcation analysis, ghost of attractor, ecological resilience.

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