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A High-Performance Hybrid Block Method for Direct Solution of Second-Order Differential Systems

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Abstract

This work introduces an advanced and optimized implicit hybrid block method developed for the direct and high-precision numerical integration of general second-order initial value problems (IVPs) [1]. The proposed method represents a substantial methodological advancement by combining the characteristics of hybrid (off-step) approaches with the computational efficiency of block methods [2]. The hybrid framework of proposed method strategically evaluates the intermediate off-step points, thereby enhancing the order of accuracy by optimizing the error constants. The developed method significantly reduces the total number of function evaluations and increases throughput compared with traditional step-by-step solvers by concurrently computing a block of solution points. In addition, the block structure of the algorithm provides a self-starting capability. The proposed algorithm supports an adaptive step-size control mechanism which is a key component of its robustness [3]. This mechanism allows the algorithm to respond to the local behavior of the solution by dynamically adjusting the integration step-size. By continuously estimating the local truncation error, the method can enlarge the step-size in regions of smooth variation to improve efficiency and decrease it in regions exhibiting sharp transitions or oscillatory behavior, thereby accurately capturing intricate solution dynamics while maintaining reliability and preventing numerical failure. Extensive numerical experiments are conducted on a comprehensive set of benchmark problems, demonstrating the computational superiority, adaptability and efficacy of the algorithm. The performance of algorithm is measured using both fixed and adaptive step-size implementations. The results show that this adaptive hybrid block method is highly robust, maintaining high accuracy and stability across a wide range of problem types, including oscillatory and stiff systems.

Keywords: ODEs, hybrid method, block method, optimization, embedded-type procedure.

References:

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