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On a q-Deformed Pythagorean Triples

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Abstract

Recent work by Mathevet, Morier-Genoud, and Ovsienko (2026) introduced a novel q-deformation of the classical Pythagoras equation, $\mathcal{A}(q)^2 + q\mathcal{B}(q)^2 = \mathcal{C}(q)\mathcal{C}^*(q)$, constructing an infinite family of polynomial solution triples systematically indexed by standard Pythagorean triples through a q-deformed action of the modular group $PSL(2,\mathbb{Z})$. While this dynamic framework successfully generates standard solutions, it leaves alternative 'exotic' solution orbits—discovered via numerical experimentation—algebraically isolated and unclassified.

In this paper, we propose an alternative, static classification paradigm by embedding the q-deformed Pythagoras equation directly into the commutative quadratic ring extension $\mathcal{R} = \mathbb{Z}[q][\sqrt{-q}]$. By reinterpreting the left-hand polynomial combination as an algebraic norm, the task of finding valid polynomial triples is reframed as a problem of characterizing principal ideals invariant under a reciprocal involution operator. Utilizing a generalized polynomial form of the Brahmagupta–Fibonacci identity, we establish the exact structural and degree-bound constraints required to satisfy the necessary monic, positive-coefficient, and self-reciprocal boundary conditions. This ring-theoretic approach provides a rigorous algebraic mechanism that accounts for the existence of non-factorable exotic solutions, mapping them to distinct polynomial orbits outside the scope of the standard modular tree. Finally, we discuss how this framework can be leveraged to address the open conjecture on coefficient unimodality using complex root distribution on the unit circle.

Keywords: q-Deformed Pythagoras equation, ring extensions, quadratic rings.

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