



Tail Asymptotics for Products of Generalized Gamma Random Variables

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Abstract

Products of random variables arise in reliability theory, wireless communications, risk modelling, and related areas of applied probability. Motivated by our previous work on products of Weibull random variables [1], we consider the product of independent generalized gamma random variables with non-identical parameters. More precisely, let

$$\Pi_n = \prod_{k=1}^n \xi_k, \quad \xi_k \sim GG(a_k, d_k, p_k), \quad k = 1, \dots, n,$$

where $GG(a, d, p)$ denotes the generalized gamma distribution with density

$$f(x) = \frac{(p/a^d)x^{d-1}e^{-(x/a)^p}}{\Gamma(d/p)}, \quad x > 0.$$

We derive a sharp asymptotic formula for the survival probability $\mathbb{P}(\Pi_n > x)$ as $x \rightarrow \infty$. The proof relies on a transformation analogous to that used in [2], the integral representation of the product tail, and the multidimensional Laplace method; see, for instance, [3].

Keywords: Generalized gamma distribution, product of random variables, Laplace method, tail distribution function.

References:

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