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Comparative Approximation Analysis of Generalized Neural Network Operators on Damped Time Series

Seda Karateke

Department of Software Engineering, Istanbul Atlas University, 34408 İstanbul, Türkiye
e-mail: seda.karateke@atlas.edu.tr

Abstract

Neural network operators form a constructive framework for quantitative approximation and numerical signal modeling. Motivated by sigmoidal neural network approximation theory [1], fractional neural network approximation [2], recent infinite-domain ordinary and fractional quasi-interpolation schemes [3], and generalized neural network operator constructions [4], this study compares three denominator-free neural network operators on noisy damped time-series data: the sampling-type S_n operator, the Kantorovich-type K_n operator, and the quadrature-type Q_n operator. The theoretical framework is based on modulus-of-continuity-type Jackson estimates and on the fractional approximation viewpoint involving left and right Caputo fractional derivatives. In the computational setting, S_n uses pointwise samples, K_n incorporates local averaging through integral means, and Q_n replaces continuous integration by weighted numerical quadrature. Numerical validation is performed on a 60-month damped harmonic oscillator signal with additive Gaussian noise. The first 48 months are used for hyperparameter selection, and the final 12 months are reserved for blind testing. The S_n operator achieves an MAE of 0.7787 and an R^2 value of 0.9518, while K_n yields an MAE of 0.8142 and an R^2 value of 0.9389. The Simpson-based Q_n operator closely reproduces the Kantorovich performance while reducing the cost of repeated integral evaluations.

These findings indicate that denominator-free neural network operators provide an efficient and theoretically grounded approximation strategy for noisy damped time-series signals.

Keywords: Neural network operators, damped time series, approximation metrics.

References:

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