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An Investigation of a Special Type-2 Fuzzy Metric

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Abstract

Many real-world problems contain not only uncertainty but also additional ambiguity about the reliability of that uncertainty. Classical mathematical models based on precise distance definitions often encounter significant limitations in practical applications due to measurement errors, perceptual inconsistencies, and unfavorable environmental conditions. Consequently, the reconsideration of fundamental notions such as proximity, similarity, and distance in uncertain environments has emerged as an important research topic from both theoretical and practical perspectives. In this regard, Type-2 fuzzy numbers, originally introduced by Zadeh [1] and subsequently developed in depth by Mendel [2], provide a powerful framework for modeling higher-order uncertainty. Unlike Type-1 fuzzy sets, where uncertainty is represented solely through membership grades, Type-2 fuzzy sets incorporate uncertainty directly into the membership functions themselves. This additional degree of freedom enables a more comprehensive representation of imprecise and ambiguous information. The concept of α-cuts, which plays a central role in the analysis of Type-1 fuzzy numbers, has been generalized to (α, β)-cuts for Type-2 fuzzy numbers. This extension has provided an effective analytical tool for investigating the structural and mathematical properties of Type-2 fuzzy numbers. In particular, approaches based on (α, β)-sections have bridged a significant gap in the literature by facilitating the systematic study of Type-2 fuzzy quantities within a rigorous mathematical framework. On the other hand, Kaleva and Seikkala [3] extended the classical notion of a metric by introducing a distance function whose values are non-negative Type-1 fuzzy numbers.

Motivated by the increasing need to model higher-order uncertainty, the primary objective of the present study is to incorporate the second-order uncertainty represented by Type-2 fuzzy sets into the metric framework originally proposed by Kaleva and Seikkala. To this end, a Type-2 fuzzy metric obtained by especially the absolute value function is constructed explicitly through (α, β)-sections, thereby providing a natural and systematic extension of classical fuzzy arithmetic to the Type-2 setting.

Keywords: Type-2 fuzzy metric, (α, β)-level sets, second-order uncertainty.

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