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On Almost A-Statistical Convergence for Double Sequences and Korovkin-Type Approximation

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Abstract

In approximation theory, classical convergence is sometimes insufficient to analyze the behavior of positive linear operators, necessitating the use of generalized summability methods. Motivated by recent developments for single sequences, the primary aim of this study is to first introduce the concept of almost A-statistical convergence for double sequences. Specifically, for a subset $F \subseteq \mathbb{N}_0^2$ and an almost regular four-dimensional matrix $A = (a_{nmij})$, we define the almost A-density of F as

$$\delta_A^{2,a}(F) := P - \lim_{p,q\to\infty} \frac{1}{pq} \sum_{n=s}^{s+p-1} \sum_{m=t}^{t+q-1} \sum_{(i,j)\in F} a_{nmij}$$

uniformly in s and t. Consequently, a double sequence $x = (x_{ij})$ is said to be almost A-statistically convergent to a limit L if for every $\epsilon > 0$, the following condition holds:

$$\delta_A^{2,a}\left(\left\{(i,j) \in \mathbb{N}_0^2 : |x_{ij} - L| \geq \epsilon\right\}\right) = 0.$$

Following this new mathematical construction, we establish a Korovkin-type approximation theorem for double sequences of positive linear operators acting on the space of continuous functions of two variables. To demonstrate the significance and effectiveness of our theoretical results, we construct an illustrative example of a sequence of positive linear operators that fails to converge in the classical or ordinary statistical sense but converges under our newly proposed method. Finally, the theoretical developments are supported by MATLAB-based numerical experiments. Graphical representations are provided to illustrate the convergence behavior of the constructed example, substantiating the mathematical claims with computational evidence.

Keywords: Almost A-summability, double sequences, Korovkin-type theorem, positive linear operators.

References:

[1] K. Demirci, F. Dirik and S. Yıldız, Almost A-statistical convergence and approximation theorems. Filomat 39 (2025), no. 23, 8117–8128.

[2] P. P. Korovkin, On convergence of linear positive operators in the space of continuous functions (in Russian). Doklady Akad. Nauk SSSR (N.S.) 90 (1953), 961–964.

[3] G. G. Lorentz, A contribution to the theory of divergent sequences. Acta Math. 80 (1948), 167–190.

[4] F. Móricz and B. E. Rhoades, Almost convergence of double sequences and strong regularity of summability matrices. Math. Proc. Cambridge Philos. Soc. 104 (1988), no. 2, 283–294.

[5] M. Mursaleen and E. Savaş, Almost regular matrices for double sequences. Studia Sci. Math. Hungar. 40 (2003), no. 1-2, 205–212.