



A Topological Characterization of Fragments via Zero Sets

Sezer Bolat⁽¹⁾ and Nazife Erkuşun-Özcan⁽²⁾

⁽¹⁾*Hacettepe University, Department of Mathematics, Ankara, Türkiye
e-mail: sezer.bolat314@gmail.com*

⁽²⁾*Hacettepe University, Department of Mathematics, Ankara, Türkiye
e-mail: erkursun.ozcan@hacettepe.edu.tr*

Abstract

Freudenthal's spectral theorem is one of the most important statements in the theory of vector lattices [1]. It fundamentally highlights the importance of investigating the fragments of positive elements within these spaces. Recent developments have begun to bridge this lattice-theoretic framework with topology, establishing initial connections between the fragment structures of continuous functions and their associated zero sets [2].

In this work, we deepen this connection by providing a direct topological characterization of fragments within the space $C[0, 1]$. Specifically, we prove that a continuous function $f \in C[0, 1]$ admits a proper fragment if and only if its zero set $Z(f)$, endowed with the subspace topology, contains a nontrivial clopen subset. Moreover, motivated by recent advancements in the complexification of vector lattices [3], we extend our topological approach to the complex function space $C[0, 1] + iC[0, 1]$. By analyzing the zero components of the real and imaginary parts, we establish explicit topological criteria for the existence of proper complex fragments.

This talk is based on an ongoing joint work with N. Erkuşun-Özcan.

Keywords: Vector lattices, continuous function spaces, fragment structure, zero set, clopen set, complexification.

References:

- [1] C. D. Aliprantis and O. Burkinshaw, Positive Operators. Springer, Dordrecht, 2006.
- [2] S. Bolat, N. Erkuşun-Özcan and N. A. Gezer, On fragments on lattice normed vector lattices. Results Math. 78 (2023), no. 6, Paper No. 245, 13 pp.
- [3] N. Dzhusoeva, J. Huang, M. Pliev and F. Sukochev, Lateral order on complex vector lattices and narrow operators. Math. Nachr. 296 (2023), no. 11, 5157–5170.