



Multiple Binomial-Type Kantorovich Operators and Their Approximation Properties

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Abstract

Binomial-type polynomial sequences, first introduced by Popoviciu [1], provide a natural setting for constructing positive linear operators. Building on this, Sablonnière [2] introduced the Bernstein-Sheffer operators and investigated their approximation properties on $C[0, 1]$. In this work, we introduce multiple Bernstein-Kantorovich operators as a Kantorovich-type extension [3] of the multiple binomial-type operators recently studied in [4]. These operators are constructed via multiple binomial-type polynomial sequences defined through the exponential generating function $e^{xH(t_1, t_2)}$, extending the classical single-index framework to a multi-index setting. We establish the moments and central moments of the operators and prove uniform convergence via the Korovkin theorem. Quantitative estimates are obtained using the classical modulus of continuity, the second-order modulus of smoothness, and Peetre's κ -functional. Approximation rates for Lipschitz-type function classes are also derived. In particular, the linear choice H yields the classical Bernstein-Kantorovich operator of degree $2n$, achieving an improved convergence rate by a factor of $1/\sqrt{2}$ compared to the classical operator of degree n . Finally, for the Bell-type generating function H , the second central moment of the multiple operator is asymptotically half that of the corresponding single-index operator, independently of any additional parameters.

Keywords: Multiple Bernstein-Kantorovich operators, modulus of continuity, Sheffer polynomials, Peetre's κ -functional, Bell numbers.

References:

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