Cesàro Summability of Fourier-Bessel series and Its Consequences

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Abstract

In this paper, we study the Cesàro summability of order δ, (C, δ), $\delta > -1$, on the Fourier–Bessel series, and using the $n^{th}-$Cesàro mean of order $'\delta'$, we approximate functions with a jump discontinuity, for which the classical Fourier–Bessel series partial sums exhibit Gibbs phenomena near the point of discontinuity and show ringing behaviour at points away from the discontinuity [1, 2]. Inspired by the classical theory of Cesàro summability for trigonometric Fourier series [3, 4], we formulate the (C, δ) summability of the Fourier–Bessel series and develop the corresponding theory and results. In particular, an explicit kernel representation analogous to the trigonometric Fourier series is introduced, and its basic properties are studied.

In the numerical study, the approximation is illustrated using a benchmark function, a step function with a jump at $x = 1/2$. A systematic comparison is shown by initially fixing the number of terms (N) in the n^{th} (C, δ)-mean and varying the summability factor δ, and then by fixing δ while varying N. The obtained results show that for sufficiently large N, Cesàro means mitigate Gibbs phenomena near discontinuities, and reduce oscillations which is present in partial sums. Moreover, we observe that as δ increases with fixed N, the graph of the Cesàro mean becomes noticeably smoother but simultaneously deviates from the true value of the function. Therefore, small positive values of δ are more effective than larger ones. Overall, these results show that the (C, δ), $\delta > -1$ means of Fourier-Bessel series provide an effective method for approximating functions, especially when the functions have jump discontinuities.

Keywords: Fourier–Bessel series, Cesàro summability, Gibbs phenomenon.

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