



The Ratio of the Generalized Marcum Functions of the First and Second Kind: Monotonicity and Bounds

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Abstract

Let $Q_\nu(a, b)$ be the generalized Marcum Q -function and $R_\nu(a, b)$ be the generalized Marcum function of the second kind of order $\nu > 0$. In this paper our aim is to present some new results related to the monotonicity and bounds on the ratio of the generalized Marcum functions of the first and second kinds $Q_\nu(a, b)/R_\nu(a, b)$. We also present new closed form series and integral representations for $R_\nu(a, b)$ and its higher order derivatives. Moreover, we show that $R_\nu(a, b)$ is infinitely divisible distribution and also belongs to the classes of self-decomposable distributions, generalized gamma convolutions and hyperbolically completely monotone densities.

Keywords: Complementary error function, modified Bessel functions, generalized Marcum Q -function, generalized Marcum function of the second kind, bounds, log-convexity and geometrical convexity, survival function.

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