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An Operator Version of the Korovkin Theorem for Modified Bernstein-Kantorovich Operators and Simultaneous Approximation

Şule Yüksel Güngör[(1)], Mehmet Ali Özarslan[(2)] and Bayram Çekim[(3)]

[(1)] *Gazi University, Department of Mathematics, Ankara, Türkiye*
e-mail: sulegungor@gazi.edu.tr

[(2)] *Eastern Mediterranean University, Department of Mathematics, Gazimagusa, TRNC Mersin 10, Türkiye*
e-mail: mehmetali.ozarslan@emu.edu.tr

[(3)] *Gazi University, Department of Mathematics, Ankara, Türkiye*
e-mail: bayramcekim@gazi.edu.tr

Abstract

In this talk, we introduce a generalized family of positive linear operators that unifies the modified Bernstein-Kantorovich operators studied by Özarslan and Duman [3] with the operator-theoretic Korovkin framework developed by Popa [4], Kiran Kumar and Vinaya [2], and Dorai [1]. Our operators map continuous functions on a closed interval into continuous functions on a compact Hausdorff space, and incorporate a shape parameter that influences both the limit operator and the rate of approximation.

We establish a uniform convergence theorem, showing that the convergence of the operator sequence to a weighted composition operator is equivalent to the uniform convergence of an underlying sequence of continuous functions. This theorem extends and incorporates several known results from the literature as special cases.

Our main contribution is a simultaneous approximation theorem, showing that not only the operators but also their derivatives of any order converge uniformly to the corresponding derivatives of the limit operator, and we characterize this simultaneous convergence as equivalent to the uniform convergence of the underlying sequence of continuous functions. The proof relies on finite difference operators, the Faà di Bruno formula, and partial Bell polynomials. We also establish quantitative estimates for the rate of convergence for functions in Lipschitz classes.

Keywords: Bernstein-Kantorovich operators, Korovkin approximation theorem, simultaneous approximation.

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