



What are the Advantages of the Granular Derivatives in Solving Fuzzy Differential Equations?

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Abstract

In this talk, we use recently proposed concepts of horizontal membership function (HMF) and granular derivative based on granular difference to introduce and study the equivalence relation between granular differential equations and a class of fuzzy differential equations (FDEs). It is well known that in the study of FDEs fuzzy interval arithmetic and differentiability concept are very important. But some restrictions imposed by the approaches which are based on these concepts make it difficult to solve FDEs. Some of restrictions associated to previous approaches—Hukuhara differentiability, generalized Hukuhara differentiability- are the existence of unnatural behavior in the modeling phenomenon, the existence of multi-solutions with different geometric presentations, the diameter of the fuzzy function be monotonic, the result of each of the four basic operations performed on fuzzy numbers is not a fuzzy number etc. So thanks to the relative-distance-measure (RDM) fuzzy interval arithmetic based on RDM variables enable direct introducing uncertain interval or fuzzy variable-values in usual mathematical formulas together with crisp values. Moreover, we compare the proposed approach with other ones based on other derivatives using some examples. In addition, we discuss the efficiency and effectiveness of the proposed approach in detail.

Keywords: Fuzzy differential equations (FDEs), granular differentiability, horizontal membership functions, relative-distancemeasure (RDM).

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