



Special Functions on Time Scales

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Abstract

We discuss a new development in the time scales polynomial shift operator, which suggests a systematic way to generalize polynomial-coefficient differential equations to dynamic equations on time scales. This facilitates the generalization of many special functions, particularly hypergeometric series, to a broad class of time scales. Historically, this was difficult due to a lack of a time scales analogue of the formula $t \sum a_k t^k = \sum a_k t^{k+1}$, but an operational formula using the Laplace transform and its inverse was suggested in 2015 to answer the question. We will discuss how that operational formula simplifies on various time scales and thereby gives us analogues of many classical special functions on time scales.

Keywords: Time scales, hypergeometric function, polynomial shift operator.

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