

ABSTRACT BOOK

ATSF 2026 CONFERENCE

International Conference:

APPROXIMATION THEORY AND SPECIAL FUNCTIONS

ATSF 2026 - 9th SERIES

SEPTEMBER 2-5, 2026

TOBB ECONOMICS AND TECHNOLOGY UNIVERSITY
ANKARA - TÜRKİYE

This is the final version of the Abstract Book.

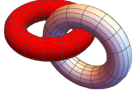
This abstract book has been published by ATSF Publishing.

Published on September 7, 2026.

© 2026 ATSF 2026 Conference. All rights reserved.

This Abstract Book is published as the official collection of abstracts presented at ATSF 2026 Conference. No part of this publication may be reproduced, distributed, republished, or made available through any other publication or electronic platform, in whole or in part, without the prior written permission of the ATSF 2026 Organizing Committee, except for brief quotations used for academic citation purposes.

The copyright of individual abstracts remains with their respective authors.



Conference Chairs

Oktay Duman (TOBB ETÜ, Türkiye)
Esra Erkus-Duman (Gazi University, Türkiye)

Invited Speakers

George A. Anastassiou (University of Memphis, USA)
Octavian Agratini (T. Popoviciu Institute of Numer. Anal., Romanian Academy, Romania)
Paolo Leonetti (Università Luigi Bocconi, Italy)
Nazife Erkuşun Özcan (Hacettepe University, Türkiye)

International Advisory and Scientific Committee

Francesco Altomare (Università degli Studi di Bari Aldo Moro, Italy)
George A. Anastassiou (University of Memphis, USA)
Martin Bohner (Missouri University of Sci. Technology, USA)
Jerry L. Bona (University of Illinois at Chicago, USA)
Francesco Dell'Accio (University of Calabria, Italy)
Alexander Goncharov (Bilkent University, Türkiye)
Weimin Han (University of Iowa, USA)
Cihan Orhan (TOBB ETU, Türkiye)
Ioan Raşa (Technical University of Cluj Napoca, Romania)
Mircea Sofonea (University of Perpignan, France)
Ferenc Weisz (Eötvös Loránd University, Hungary)

Local Organizing Committee

Rabia Aktaş Karaman (Ankara University)
İsmail Aslan (Hacettepe University)
Nejla Özmen (Düzce University)
Ceylan Turan Yalçın (Çankaya University)
T. Yeliz Gökçer Ellidokuz (Bahçeşehir University)
Düriye Korkmaz Düzgün (Ankara Hacı Bayram Veli University)
Meryem Ece Alemdar (TOBB ETÜ)

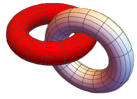
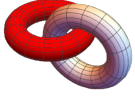


Table of Contents

• Preface	page 4
• Invited Talks	pages 12–15
• Contributed Talks	pages 16–122
• Poster Presentations	pages 123–131



Preface

The International Conference on **Approximation Theory and Special Functions (ATSF)** is a conference series devoted to bringing together researchers from different areas of approximation theory, special functions, and related fields in order to exchange new ideas, present recent developments, and foster scientific collaboration.

The ATSF series was initiated in 2013 and has since developed into a well-established international meeting, bringing together researchers from many countries and institutions around the world. Following eight successful editions held in different European cities and in Türkiye, **ATSF 2026** marks the **9th edition of the series** and is hosted by TOBB Economics and Technology University in Ankara, Türkiye, on September 2-5, 2026.

ATSF 2026 brings together **141 scientists from 20 countries**, reflecting the increasingly international character of the conference series. The scientific program covers a broad range of topics including approximation theory, special functions, summability theory, analysis, numerical methods, applied mathematics, mathematical modelling, and related areas.

The scientific program of ATSF 2026 consists of a total of **121 presentations**, including **4 invited talks**, **108 contributed presentations**, and **9 poster presentations**. This Abstract Book brings together these contributions in a single volume and provides a concise record of the scientific content of the conference.

We sincerely thank all invited speakers, participants, special session organizers, session chairs, members of the scientific and organizing committees, and all colleagues whose contributions have made ATSF 2026 possible. We are particularly grateful to **TOBB Economics and Technology University (TOBB ETÜ)** for hosting the conference and for its warm hospitality. We also gratefully acknowledge **The Scientific and Technological Research Council of Türkiye (TÜBİTAK)** and the **Turkish Academy of Sciences (TÜBA)** for their generous support of ATSF 2026.

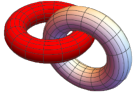
We hope that ATSF 2026 will provide a stimulating environment for fruitful scientific discussions, new collaborations, and further developments in approximation theory, special functions, and related areas.

Prof. Dr. Oktay Duman
TOBB ETÜ
Department of Mathematics
Conference Co-Chair

Prof. Dr. Esra Erkuş-Duman
Gazi University
Department of Mathematics
Conference Co-Chair

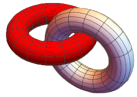
The ATSF 2026 Conference is partially supported by:





Invited Talks

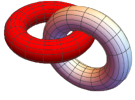
- George A. Anastassiou.....page 12
“Multi-Composite General Neural Network Approximation Over Finite Dimensional Banach Spaces”
- Octavian Agratini.....page 13
“Study of Qualitative Properties of Some Linear Approximation Processes”
- Paolo Leonetti.....page 14
“Ideal Core of Real Sequences and Linear Transformations”
- Nazife Erkuşun Özcan.....page 15
“Positivity and Asymptotic Behavior of Operator Semigroups”



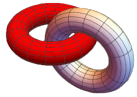
Contributed Talks

Sorted by presenting author name

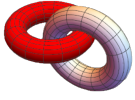
- Adnan Khan page 16
“Three-Dimensional Laplace Transform Coupled with the Three-Dimensional Marichev Saigo Maeda Integral Operator and Generalized Incomplete Hypergeometric Function”
- Ahmed Sani page 17
“The Maximal Regularization of Smooth Copulas”
- Alexander Goncharov page 18
“Whitney Spaces Without the Bounded Approximation Property”
- Alexander Kushpel page 19
“Widths of Sets of Smooth Functions on Two-Point Homogeneous Spaces”
- Alper Erdem page 20
“On Certain Orbital Convergences with an Arzelà-Ascoli Type Theorem”
- Asiye Arif page 21
“Approximation by Neural Network Operators of Convolution Type Activated by Deformed and Parametrized Half Hyperbolic Tangent Function”
- Astrit Ferizi page 22
“Shearlet Expansions and Asymptotic Behavior of Lizorkin Distributions”
- Aynura Poladova page 23
“Approximate Formulas for Boundary Functionals of Random Walk Process”
- Aysel Özfidan page 24
“Exact Solutions of the s -Wave Klein-Gordon Equation With the Squared-Cotangent Potential”
- Aysel Özfidan page 25
“Relativistic Treatment of Spin-0 Particles Under the Trigonometric Scarf Potential”
- Aytekin Enver page 26
“Fractional Reaction-Diffusion Modeling and Numerical Simulation of DNA Sequence Dynamics Using a Unified FD-FEM Framework”
- Bahriye Karaca page 27
“Fixed Point Methods for Nonlinear Boundary Value Problems in Complex Spaces”
- Barış Can Çakır page 28
“A Fibonomial Version of Truncated-Exponential-Based Appell-Type Changhee Polynomials”
- Benaissa Zerroudi page 29
“A Multinode Shepard Framework for the Numerical Solution of Integral Equations”
- Berke Şahin page 30
“Approximation Properties of Multivariate Max-Min Neural Network Operators of Durrmeyer Type”
- Betül Hiçdurmaz page 31
“Exponential Trapezoidal Product Integration Schemes on Graded Meshes for a Tempered Type Integrodifferential Equation”
- Betül Hiçdurmaz page 32
“Polynomial and Product Integration Approximations for Volterra Integral Equations with Tempered Kernels”
- Brahimi Mahmoud page 33
“2D Nonstationary Wavelet Filter Design via Bézout and Sum-of-Squares Decomposition”
- Bülent Köroğlu page 34
“Converses of Diamond Alpha Pachpatte Type Dynamic Inequalities and Their Applications in Oscillation of Self Adjoint Linear Dynamic Equations”
- Cemaliye Kürt page 35
“A Study on Tempered Fractional Calculus Involving the Multivariate Mittag-Leffler Function”
- Ceylan Turan Yalçın page 36
“From Convergence Theory to Data Analysis: Is a New Approach Possible?”



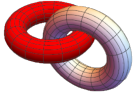
• Chaimaa Benzarouala	page 37
“Fixed Point Results in Generalized Rectangular Metric Spaces in the Sense of Branciari and Their Applications”	
• Dilara Karshoğlu	page 38
“Strict Hölder Stability of Perturbed Nonlinear Matrix Differential Systems with Initial Time Difference”	
• Ece Özdemir	page 39
“On the Isospectrality Relation Between PT-Symmetric Schrödinger Operators and Their Adjoints”	
• Ecem Acar	page 40
“Approximation in Weighted Spaces by Bivariate Max-Product Operators”	
• El Amir Djefal	page 41
“Volterra Integral Equation Framework for Optimal Drug Administration in Tumor Growth Control”	
• Elif Naz Fidancı	page 42
“Enhancing Feature Selection in Molecular Dynamics Trajectory Data for Hidden Markov Models”	
• Ercan Gürvit	page 43
“Fluctuationlessness Theorem Applied to Caputo Fractional Initial Value Problems”	
• Esra Çekirdek	page 44
“Fourier Transforms of Orthogonal Polynomials on The Double Cone”	
• Esra Gündoğan Lekesiz	page 45
“A Novel Set of Univariate Biorthogonal Polynomials”	
• Esra Karaoğlu	page 46
“Operational Properties of Two-Parameter Laguerre-Type Sheffer Polynomials”	
• Ezgi Alkan	page 47
“ k -Hypergeometric Functions and Applications”	
• Ezgi Han Eryüksel	page 48
“Asymptotic Behavior of Sequences in Vector Lattices”	
• Fatih Yılmaz	page 49
“On the Pseudospectra of q -Quasinormal Operators”	
• Fatma Güler Eroğlu	page 50
“A Time-Filtered VMS-POD Reduced-Order Model for Double-Diffusive Darcy-Brinkman Equations”	
• Fatma Yeşil Baran	page 51
“From Infinite Arrays to Finite Matrices: The Truncated Riordan Group Approach”	
• Feridun Tasdan	page 52
“Smoothed Rank-Based Regression Estimation Using Wilcoxon Score Functions”	
• Gencay Oğuz	page 53
“A Maximal Ergodic Lemma for q -Weighted Ergodic Averages”	
• Gheorghe Bucur	page 54
“Dunford-Pettis vs. Eberlein-Šmulian Theorems”	
• Gökçe Çaylak Kayaturan	page 55
“Optimizing Graph Path Encoding with Probabilistic Cryptography”	
• Güher Gülçehre Özbey	page 56
“On the Spectral and Scattering Properties of Generalized Eigenparameter-Dependent Impulsive Quantum Sturm–Liouville Equation”	
• Gülen Başcanbaz-Tunca	page 57
“On Riemann-Liouville Type Fractional Generalized Kantorovich Operators”	
• Güler Başak Öznur	page 58
“Spectral Properties of a Discrete Impulsive Schrödinger Equation”	
• Hamdullah Başaran	page 59
“Improvement of Berezin Radius Inequalities on Reproducing Kernel Hilbert Spaces by Applying Cartesian Comparability”	



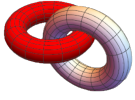
• Harun Karşlı	page 60
“Non-Isotropic Approximation by Multidimensional Mellin-Type Singular Integral Operators: Asymptotic and Quantitative Results”	
• Hasen Mekki Öztürk	page 61
“A Dynamical Systems Approach to a Climate-Driven Aquatic Ecosystem with Fisheries Management”	
• Hasen Mekki Öztürk	page 62
“Delay-Induced Instability and Chaos in a Plankton-Fish Model Regulated by Toxins, Fear and Refuge”	
• Hüseyin Erhan Altın	page 63
“On the Approximation of Borel Derivatives by Nonlinear Durrmeyer Type Operators”	
• Ileana Bucur	page 64
“Arithmetic and Measure Theory”	
• İrem Eroğlu	page 65
“Hardy-Rogers Type Fixed Point Results for Quantale-Valued Quasi-Metric Spaces”	
• İsmail Aslan	page 66
“Multidimensional Max-Min Sampling Operators with Applications”	
• Krzysztof Zakrzewski	page 67
“A Generalization of Banach’s Closed Range Theorem”	
• Mahdiah Farzin Asanjan	page 68
“ ℓ_0 -Regularized Random Graphical Modeling for Sparse Microbiome Network Inference on AGP and HMP Data”	
• Majda El Allali	page 69
“Consistent Parameter Estimation for Stochastic Differential Equations via Theta-Milstein GMM: Theory, Stability Thresholds, and Empirical Validation”	
• Manal Seddar	page 70
“Existence Results for a Neumann Problem with Hardy-Littlewood-Sobolev Critical Exponent and Hardy Potential”	
• Mehmet Ali Özarslan	page 71
“Jakimovski-Leviatan Type Operators Including Bessel’s K Function in the Kernel”	
• Mehmet Küçükaslan	page 72
“On the Space of Sequential Modulus Approximately Continuous Function”	
• Melike Berna Özkan	page 73
“Approximate Formula for Cramér Coefficient in a Non-Linear Insurance Model Under Gamma-Distributed Claims”	
• Messaoud Boulbrachene	page 74
“On the Finite Element Approximation in the L^∞ - Norm of a Class of Semi-Linear Elliptic Systems of QVIs”	
• Mohammad Oumoucha	page 75
“Spline-Based Laplace-Stieltjes Approximation for Saturated Generalized-Gamma Aggregates”	
• Mohammed Khellouf	page 76
“ $\mathcal{L}^2$ -Consistent and Asymptotically Normal Quasi-Maximum Likelihood Estimators for Milstein-Discretized Scalar Diffusion Process”	
• Moncef Zarrouk	page 77
“Approximation Laws for Optimal Stopping Under Generalized-Gamma Uncertainty”	
• Muhammad Muddassar	page 78
“Operator-Theoretic Extensions of Hermite-Hadamard Type Inequalities”	
• Muhammed Syam	page 79
“An Accurate Method for Solving Fractional Integro-Differential Equations”	
• Mustafa Gülfırat	page 80
“A Characterization of Distributional Convergence with Respect to Power Series Methods”	
• Mustafa Kara	page 81
“Szász-Durrmeyer Type Degenerate Hermite Operators”	



• Mustafa Mullahasanoğlu	page 82
“Hypergeometric Sum/Integral Transformations: Bridging Gauge Theories and Spin Models”	
• Nahed Krir	page 83
“Generalized Weinstein-Sobolev-Gevrey Spaces”	
• Nahed Krir	page 84
“Linear Canonical Fourier-Bessel Gabor Transform and Extremal Functions”	
• Nedjem Eddine Ramdani	page 85
“Oscillation Criteria for Difference Equations with Non-Monotone Deviating Arguments of Advanced Type”	
• Nelson Kiprono Bii	page 86
“Estimating a Finite Population Mean Using Transformed Data in Presence of Random Non-Response”	
• Neslihan Biricik Hepsisler	page 87
“A New Generalization of Touchard-Based Appell Polynomials”	
• Niyaz Tokmagambetov	page 88
“Growth of the Laguerre Transform in Terms of Modulus of Continuity”	
• Niyazi Anıl Gezer	page 89
“RUC Semigroups and Their Asymptotic Limit Sets”	
• Övgü Gürel Yılmaz	page 90
“On the Image of the Limit q -Stancu Operator”	
• Özge Ada	page 91
“A Class of Generalized Gegenbauer Polynomials”	
• Pınar Baydemir Daştan	page 92
“Dynamics of Oxygen-Phytoplankton Interactions: Understanding Global Warming through the Allee Effect”	
• Raymond Calvin Ochieng	page 93
“On a q -Deformed Pythagorean Triples”	
• Reyhan Canatan İlbey	page 94
“Gamma Type Operators that Preserve Logarithmic Functions”	
• Ricardas Kamarauskas	page 95
“Tail Asymptotics for Products of Generalized Gamma Random Variables”	
• Seda Karateke	page 96
“Comparative Approximation Analysis of Generalized Neural Network Operators on Damped Time Series”	
• Selma Negzaoui	page 98
“Donoho-Stark Uncertainty Principles in the Laguerre Hypergroup Setting”	
• Semra Kuş	page 99
“Degenerate Sheffer F -Polynomials and Their Properties”	
• Sena Nur Sancak	page 100
“Generalization of Jakimovski-Leviatan-Păltănea Operators Based on Special Polynomials”	
• Sevda Yıldız	page 101
“On Almost A -Statistical Convergence for Double Sequences and Korovkin-Type Approximation”	
• Sezer Bolat	page 105
“A Topological Characterization of Fragments via Zero Sets”	
• Sezin Çit	page 106
“Multiple Binomial-Type Kantorovich Operators and Their Approximation Properties”	
• Shailesh Kumar Srivastava	page 107
“On Cesàro Summability of Fourier-Bessel series and Its Consequences”	
• Sihem Djellab	page 108
“DFT-Based Computational Prediction of Hypothetical Cubic C1b SrMgSi and SrMgSn Half-Heusler Phases”	
• Simanur Başaran	page 109
“On The Analytical Properties of The Telephone Polynomials”	



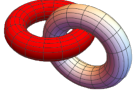
• Sourav Mandal.....	page 110
“On Rough Topological Spaces”	
• Sumeyra Ucar.....	page 112
“Fractional-Order Mathematical Model of Genetically Predisposed Diabetes Mellitus and Graphical Representation of Model Classes”	
• Şehla Eminoğlu.....	page 97
“An Investigation of a Special Type-2 Fuzzy Metric”	
• Şeyda Sezgek.....	page 102
“On Cesàro Submethods and Statistical Convergence in Neutrosophic Normed Spaces”	
• Şeyma Bilazeroğlu.....	page 103
“Dynamical Analysis of a Leslie-Gower Predator-Prey Model with Fear Effect and Prey Refuge”	
• Şeyma Çelik.....	page 104
“Some Applications of Power Series Statistical Convergence”	
• Şule Yüksel Güngör.....	page 111
“An Operator Version of the Korovkin Theorem for Modified Bernstein-Kantorovich Operators and Simultaneous Approximation”	
• Tahir Ceylan.....	page 114
“What are the Advantages of the Granular Derivative in Solving Fuzzy Differential Equations?”	
• Tatiana Filippova.....	page 115
“Approximation Approach in State Estimation Problems for Dynamical Control Systems Under Uncertainty”	
• Tülay Yazır.....	page 116
“An Approximate Formula for the Expected Value of Residual Waiting Time Process Under Heavy-Tailed Distributions”	
• Türkan Yeliz Gökçer Ellidokuz.....	page 113
“Max-Product Shepard-Lidstone Operators and Their Approximation Properties”	
• Ulrich Reif.....	page 117
“New Enclosures for Bessel Functions”	
• Yahya Çin.....	page 118
“On a New Class of Multivariable k -Gottlieb Polynomials and Their Analytical Properties”	
• Yasin Ünsal.....	page 119
“A New Generalization of Truncated-Type Polynomials and Their Properties”	
• Zakaria El Allali.....	page 120
“A Deformation-Adaptive First-Order Optimization Framework Using Jackson q -Gradients”	
• Zehra Nur Öztürk.....	page 121
“Investigation of Hyperbolic Cahn-Hilliard Equations with Dynamic Boundary Conditions”	
• Zeynep Özat.....	page 122
“On Appell Polynomials Associated with k -Hypergeometric Functions and Their Properties”	



Poster Presentations

Sorted by presenting author name

- Ashi Taşkan page 123
“Approximation by Generalized Abel-Poisson Means of Hexagonal Fourier Series”
- Emine Merve Arslan.....page 124
“Approximation by Deferred Means of Hexagonal Fourier Series”
- Emre Taş.....page 125
“Some Approximation Results for Lototsky Type Bernstein-Schurer Operators”
- Imene Khélifa page 126
“Determination of a Radially Symmetric Potential in an Inverse Scattering Problem”
- Karima Kimouche page 127
“Parameters Estimation of a Spatio-temporal ARCH Model with Location Dependent Parameters”
- Lorentz Jäntschi.....page 128
“Outlier Detection in the Approximation of Tension Strain Energy of C_{44} Fullerenes: An Approach Based on Fragmental Matrix Property Indices and Non-Parametric Residual Analysis”
- Nimete Berisha.....page 129
“On Fourier transforms of Confluent Hypergeometric Functions and Ramanujan-Type Formulas”
- Sofiane Ouazine page 130
“Numerical Approach for the Analysis and Optimization of GI/M/1/N Queue with Vacations”
- Tuğba Yurdakadim.....page 131
“Approximation Results for Chlodowsky Modification of A New Bernstein-like Operators”



(Invited Talk)

Multi-Composite General Neural Network Approximation Over Finite Dimensional Banach Spaces

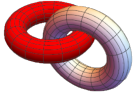
George A. Anastassiou

*Department of Mathematical Sciences, University of Memphis, Memphis, USA
e-mail: ganastss@memphis.edu*

Abstract

In this talk, we consider functions defined on a finite-dimensional Banach space of dimension $N \in \mathbb{N}$ and taking values in $\mathbb{R}^N$. By exploiting suitable topological properties of this setting, we develop a multivariate multi-composite general neural network approximation scheme for such functions. Our treatment is quantitative in nature. In particular, we establish multivariate multi-composite Jackson-type inequalities in terms of the modulus of continuity of the function under approximation. The resulting convergence is obtained both pointwise and uniformly. Moreover, the method is expected to yield accelerated rates of convergence.

Keywords: Finite dimensional Banach spaces, multi-composite general neural network operators approximation, modulus of continuity, multicomposite general accelerated approximation.



(Invited Talk)

Study of Qualitative Properties of Some Linear Approximation Processes

Octavian Agratini

Tiberiu Popoviciu Institute of Numerical Analysis, Romanian Academy, Cluj-Napoca, Romania
e-mail: o.agratini@yahoo.com

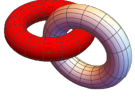
Abstract

It is acknowledged that linear positive operators are a useful tool in approximation signals from various spaces. In our talk we propose to analyze qualitative properties in three directions.

Referring to operators of either discrete or continuous type, a natural challenge is to highlight the asymptotic expansion valid for locally smooth functions. All coefficients are derived and explicitly given. Each time a Voronovskaja type theorem is reobtained. Our presentation focuses primarily on this aspect. Here are six classes of operators studied from this point of view: the semi-exponential Post–Widder operator with the power function p , $p(x) = x^2$, $x \geq 0$; a general class of convolution operators based on Landau functions enjoying a feature less commonly encountered by integral type operators, namely they reproduce the affine functions; an extension of Weierstrass integral operators correlated with the diffusion equation and involving modified Bessel functions; a class of higher order Szász–Mirakjan operators of Kantorovich type; Jain operators based on a generalized Poisson distribution; the quasi-interpolants of Gauss–Weierstrass operators, and our results apply to all integrable real functions f on $\mathbb{R}$ satisfying the growth condition $f(t) = \mathcal{O}(\exp(ct^2))$ as $|t|$ tends to ∞ for some $c > 0$.

The second direction of study is about uniform approximation of some classes of linear positive operators expressed by series. It is known that the Bohman–Korovkin criterion does not guarantee uniform convergence for operators acting on functions defined on unbounded intervals. In our talk we identify functions for which the operators provide uniform approximation over unbounded intervals.

The last part offers a transition from uniform to statistical convergence. Our study pays special attention to binomial operators.



(Invited Talk)

Ideal Core of Real Sequences and Linear Transformations

Paolo Leonetti^{(1),(2)}

⁽¹⁾ *Università degli Studi dell'Insubria, Varese, Italy*

⁽²⁾ *Università Bocconi, Milano, Italy*

e-mail: leonetti.paolo@gmail.com

Abstract

Given an ideal $\mathcal{I}$ on the natural numbers ω and a bounded real sequence $\mathbf{x}$, we denote by

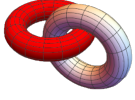
$$\text{core}_{\mathbf{x}}(\mathcal{I})$$

the smallest interval $[a, b]$ such that $\{n \in \omega : x_n \notin [a - \varepsilon, b + \varepsilon]\} \in \mathcal{I}$ for all $\varepsilon > 0$. Note that, if $\mathcal{I}$ is the ideal Fin of finite subsets of ω , then

$$\text{core}_{\mathbf{x}}(\text{Fin}) = [\liminf \mathbf{x}, \limsup \mathbf{x}].$$

In addition, $\text{core}_{\mathbf{x}}(\mathcal{I}) = \{\eta\}$ if and only if $\mathcal{I}\text{-}\lim \mathbf{x} = \eta$.

Thus, given an infinite matrix $A = (a_{n,k} : n, k \in \omega)$ and ideals $\mathcal{I}, \mathcal{J}$ on ω , we provide several relationships between the $\mathcal{I}$ -core of $\mathbf{x}$, the $\mathcal{J}$ -core of the linear transformation $A\mathbf{x}$, and certain regularity properties of A . Incidentally, we show that the unit ball of $\ell_{\infty}(\mathbf{R}^d)$ coincides with the closed convex hull of its extreme points.



(Invited Talk)

Positivity and Asymptotic Behavior of Operator Semigroups

Nazife Erkurşun Özcan

Hacettepe University, Department of Mathematics, Ankara, Türkiye
e-mail: erkursun.ozcan@hacettepe.edu.tr

Abstract

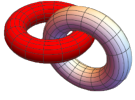
Positive operators and semigroups play an important role in several branches of modern analysis, including approximation theory, ergodic theory, evolution equations, and dynamical systems. In many situations, the long-term dynamics of operator semigroups are strongly influenced by the underlying structure of the spaces on which they act, making positivity an important tool in asymptotic analysis.

This talk focuses on operator semigroups on ordered Banach spaces and Banach lattices, where order-theoretic methods provide a flexible framework extending beyond classical norm-topological approaches. We discuss how the underlying order structure influences continuity, stability, and convergence phenomena associated with operator dynamics.

Special emphasis will be placed on recent developments concerning semigroup methods in ordered settings, including extensions of classical results and new perspectives arising from generalized approaches to continuity and convergence.

The aim of the talk is to illustrate how ordered structures shape the asymptotic behavior of operator semigroups and to emphasize the role of positivity in extending semigroup-theoretic methods beyond classical settings.

Keywords: Positive operators, operator semigroups, asymptotic behavior, ordered vector spaces, Banach lattices.



Three-Dimensional Laplace Transform Coupled with the Three-Dimensional Marichev Saigo Maeda Integral Operator and Generalized Incomplete Hypergeometric Function

Adnan Khan⁽¹⁾ and Frasad Iqbal⁽²⁾

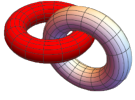
⁽¹⁾ *National College of Business Administration & Economics, Department of Mathematics,
Lahore Pakistan
e-mail: adnankhantariq@ncbae.edu.pk*

⁽²⁾ *National College of Business Administration & Economics, Department of Mathematics,
Lahore Pakistan
e-mail: ghsfrasatiqbal@gmail.com*

Abstract

In this paper, a three-dimensional (3D) Laplace transform is studied in conjunction with the Marichev-Saigo-Maeda (MSM) fractional integral operators and the generalized incomplete hypergeometric function. The proposed framework extends existing results by incorporating both left- and right-sided Saigo-Maeda operators, providing a unified approach for analyzing fractional integral transformations in higher dimensions. Several new results and formulations are derived, demonstrating the interaction between the 3D Laplace transform and MSM operators applied to incomplete hypergeometric functions. Furthermore, special cases and corollaries are presented to highlight the applicability and generalization of the obtained results. The proposed approach offers a new perspective in symmetry analysis and contributes to the advancement of fractional calculus with potential applications in mathematical physics and related fields.

Keywords: Integral transformation, Laplace transformation, operators, left & right sided SM operator, integrals, hypergeometric function.



The Maximal Regularization of Smooth Copulas

Ahmed Sani⁽¹⁾, M. Elmaazouz⁽²⁾ and R. Jaafar⁽³⁾

⁽¹⁾ *Ibn Zohr University, Department of Mathematics, Agadir, Morocco*
e-mail: a.sani@uiz.ac.ma

⁽²⁾ *CRMEF Center, Department of Mathematics, Inezgane, Morocco*
e-mail: maaz71@gmail.com

⁽³⁾ *Ibn Zohr University, Department of Mathematics, Agadir, Morocco*
e-mail: rachid.jaafar.59@edu.uiz.ac.ma,

Abstract

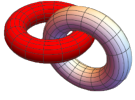
The regularity of special functions, particularly that of copulas, requires careful treatment, not only because this type of function describes complex inter-dependencies, but also because the mastery of prediction tools depends intimately on the class of analytic (or algebraic) regularity of the mathematical model that describes both the correlation and the evolution [1]. We plan to provide some answers to this crucial problem in many fields.

For a rapid familiarization with copulas, we refer to the famous book of Nelesen [2], and for recent developments on this topic, see [3] for construction process of such functions or [4] for emerging applications in life sciences .

Keywords: Copulas, regularization, approximation.

References:

- [1] O. Elamrani, M. El Maazouz and A. Sani, An Analytic treatment of evolution copulas. *Asia Pac. J. Math.* 11 (2024), 48, 13 pp.
- [2] R. B. Nelsen, *An Introduction to Copulas*. Springer Ser. Statist. Springer, New York, 2006
- [3] O. Elamrani and A. Sani, A procedure of perturbation leading to a new class of asymmetric copulas. *Mathematics* 14 (2026), no. 7, 1093. 29 pp.
- [4] R. Jaafar, A. Hfa and A. Sani, Mathematical analysis of copulas in life sciences: An application to cardiovascular system. *Preprints* 2026, 21 pp.



Whitney Spaces Without the Bounded Approximation Property

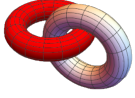
Alexander Goncharov

Bilkent University, Department of Mathematics, Ankara, Türkiye
e-mail: goncha@fen.bilkent.edu.tr

Abstract

Using Grothendieck's approach, we present a family of Whitney jet spaces that do not have the bounded approximation property.

Keywords: The bounded approximation property, Whitney jet spaces.



Widths of Sets of Smooth Functions on Two-Point Homogeneous Spaces

Alexander Kushpel

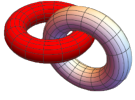
*Çankaya University, Department of Mathematics, Ankara, Türkiye
e-mail: kushpel@cankaya.edu.tr*

Abstract

Sharp orders of Cubature and Kolmogorov widths of Sobolev classes on compact homogeneous manifolds are established. In particular, we give solution of the problem of Laplace and Legendre on the d -dimensional sphere. The proof is based on new results in geometry of Banach spaces [1–7].

References:

- [1] B. Bordin, A. K. Kushpel, J. Levesley and S. A. Tozoni, Estimates of n -widths of Sobolev's classes on compact globally symmetric spaces of rank one, *J. Funct. Anal.* 202 (2003), no. 2, 307–326.
- [2] A. K. Kushpel, Optimal distribution of data points on S^2 and approximation of Sobolev's classes in $L_\infty(S^2)$. *J. Concr. Appl. Math.* 2 (2004), no. 1, 77–89.
- [3] A. K. Kushpel, Optimal cubature formulas on compact homogeneous manifolds. *J. Funct. Anal.* 257 (2009), no. 5, 1621–1629.
- [4] A. K. Kushpel and K. Taş, The radii of sections of origin-symmetric convex bodies and their applications. *J. Complexity* 62 (2021), Paper No. 101504, 21 pp.
- [5] A. K. Kushpel, Lower bounds of cointersections and widths of multiplier operators. *J. Complexity* 69 (2022), Paper No. 101614, 23 pp.
- [6] A. Kushpel, Optimal recovery and volume estimates, *J. Complexity* 79 (2023), Paper No. 101780, 15 pp.
- [7] A. K. Kushpel, John–Löwner ellipsoids and entropy of multiplier operators on rank 1 compact homogeneous manifolds. *Mat. Sb.* 216 (2025), no. 2, 81–109; translation in *Sb. Math.* 216 (2025), no. 2, 210–238



On Certain Orbital Convergences with an Arzelà-Ascoli Type Theorem

Alper Erdem

Mersin University, Department of Mathematics, Mersin, Türkiye
e-mail: alpererdem@mersin.edu.tr

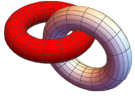
Abstract

This study considers orbital versions of alpha convergence and the notion of exhaustiveness for sequences of functions defined between metric spaces and examines their connections with existing notions. Moreover an Arzelà-Ascoli type theorem is established, showing that orbital exhaustiveness is sufficient instead of equicontinuity in the conditions to obtain an orbitally alpha convergent subsequence.

Keywords: Function sequences, alpha convergence, exhaustiveness, Arzelà-Ascoli theorem.

References:

- [1] C. Arzelà, Sulle funzioni di linee. Mem. Accad. Sci. Ist. Bologna Cl. Sci. Fis. Mat. 5 (1895), no. 5, 55–74.
- [2] G. Ascoli, Le curve limite di una varietà data di curve. Atti della R. Accad. Dei Lincei Memorie della Cl. Sci. Fis. Mat. Nat. 18 (1883), no. 3, 521–586.
- [3] R. Courant, Ueber eine Eigenschaft der Abbildungsfunktionen bei konformer Abbildung. Nachrichten von der Gesellschaft der Wissenschaften zu Göttingen. Mathematisch-Physikalische Klasse, 1914, 101–109.
- [4] V. Gregoriades and N. Papanastassiou, The notion of exhaustiveness and Ascoli-type theorems. Topology Appl. 155 (2008), no. 10, 1111–1128.



Approximation by Neural Network Operators of Convolution Type Activated by Deformed and Parametrized Half Hyperbolic Tangent Function

Asiye Arif⁽¹⁾ and Tuğba Yurdakadim⁽²⁾

⁽¹⁾*Bilecik Şeyh Edebali University, Department of Mathematics, Türkiye
e-mail: arif_asiya@mail.ru*

⁽²⁾*Bilecik Şeyh Edebali University, Department of Mathematics, Türkiye
e-mail: tugba.yurdakadim@bilecik.edu.tr*

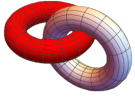
Abstract

The idea behind neural networks and artificial neural networks is to act like human brain by understanding the biological nature and function of it and they lie in the centre of machine learning. Recently, the power of computers, accessing the big data, studies on neural networks gain speed since they give us opportunity to handle some complex duties such as image recognition, natural language processing, optimization, forecasting, approximation of functions and curve fitting. Quantitative approximation of positive linear operators to the unit has been studied by G. A. Anastassiou and many mathematicians. In the present study, we introduce three kinds of convolution neural network operators with the kernel based on symmetrized, q -deformed and β -parametrized half hyperbolic tangent function. It is noteworthy to mention that it is frequently used in neural networks and they can be interpreted as positive linear operators, thus we use the methods of positive linear operator theory. We obtain quantitative convergence results to the identity operator by using modulus of continuity and global smoothness preservation of our operators are also examined. Furthermore, iterated versions of them are taken into consideration.

Keywords: Half hyperbolic tangent function, convolution type, positive linear operators, rate of convergence, iterated approximation.

References:

- [1] G. A. Anastassiou, Parametrized hyperbolic tangent based Banach space valued multivariate multi layer neural network approximations. *J. Comput. Anal. Appl.* 31 (2023), no. 4, 490–519.
- [2] G. A. Anastassiou, *Intelligent Computations: Parametrized, Deformed and General Neural Networks*. Springer, New York, Heidelberg, 2023.
- [3] R. Bojanic and O. Shisha, On the precision of uniform approximation of continuous functions by certain linear operators of convolution type. *J. Approx. Theory* 8 (1973), 101–113.
- [4] H. S. Jung and R. Sakai, Local saturation of a positive linear convolution operator. *J. Inequal. Appl.* 2014, 2014:329, 16 pp.
- [5] G. Moldovan, Discrete convolutions and linear positive operators. I. *Ann. Univ. Sci. Budapest. Eötvös Sect. Math.* 15 (1972), 31–44.
- [6] G. Moldovan, Discrete convolutions in connection with functions of several variables and positive linear operators. *Studia Univ. Babeş-Bolyai Ser. Math.-Mech.* 19 (1974), no. 1, 51–57.
- [7] A. I. Kamzolov, The order of approximation of functions of class $Z_2(E_n)$ by positive linear convolution operators (Russian). *Mat. Zametki* 7 (1970), 723–732.
- [8] J. J. Swetits and B. Wood, Local L_p -saturation of positive linear convolution operators. *J. Approx. Theory* 34 (1982), no. 4, 348–360.



Shearlet Expansions and Asymptotic Behavior of Lizorkin Distributions

Astrit Ferizi⁽¹⁾ and Katerina Hadzi-Velkova Saneva⁽²⁾

⁽¹⁾ *University of Prishtina, Faculty of Mathematics and Natural Sciences, Prishtina, Kosovo*
e-mail: astrit.ferizi@uni-pr.edu

⁽²⁾ *Ss. Cyril and Methodius University in Skopje, Faculty of Electrical Engineering and Information Technologies, Skopje, North Macedonia*
e-mail: saneva@feit.ukim.edu.mk

Abstract

While wavelets are effective for detecting the singular support of functions and distributions, their isotropic nature limits their ability to capture finer geometric features of singularities, such as those encoded by the wavefront set. To overcome these limitations and extend the advantages of wavelet methods to higher dimensions, several directional frameworks have been developed. Among them, shearlet systems, introduced by Guo, Kutyniok and Labate, provide an effective approach due to their strong analytical and geometric properties [1–3].

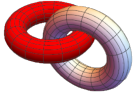
Following recent developments, the shearlet transform has been extended, via duality, to Lizorkin-type spaces of test functions and distributions, $\mathcal{S}_0(\mathbb{R}^2)$ and $\mathcal{S}'_0(\mathbb{R}^2)$ [4]. In our recent work [5], we develop a shearlet expansion theory in these spaces. We prove that shearlet series associated with Parseval systems converge in the corresponding topologies and obtain a topological characterization of Lizorkin distributions in terms of their shearlet coefficients. This framework also provides descriptions of bounded sets and weakly convergent nets via shearlet coefficients.

Finally, we apply the developed theory to characterize the quasiasymptotic behavior and quasiasymptotic boundedness of Lizorkin distributions through the asymptotic behavior of their shearlet coefficients.

Keywords: Parseval shearlet, shearlet coefficients, discrete shearlet transform, Tauberian theorems, asymptotic behavior, Lizorkin-type spaces of test functions and distributions.

References:

- [1] G. Kutyniok and D. Labate, Shearlets: Multiscale Analysis for Multivariate Data. Appl. Numer. Harmon. Anal., Birkhäuser/Springer, New York, 2012.
- [2] K. Guo and D. Labate, Optimally sparse multidimensional representation using shearlets. SIAM J. Math. Anal. 39 (2007), no. 1, 298–318.
- [3] G. Kutyniok and D. Labate, Resolution of the wavefront set using continuous shearlets. Trans. Amer. Math. Soc. 361 (2009), no. 5, 2719–2754.
- [4] F. Bartolucci, S. Pilipović and N. Teofanov, Continuity properties of the shearlet transform and the shearlet synthesis operator on the Lizorkin type spaces. Math. Nachr. 295 (2022), no. 12, 2318–2337.
- [5] A. R. Ferizi and K. Hadzi-Velkova Saneva, Shearlet expansion theory on Lizorkin-type spaces. Funct. Anal. Appl. 60 (2026), no. 1, 82–96.



Approximate Formulas for Boundary Functionals of Random Walk Process

Zulfiye Hanalioglu⁽¹⁾, Aynura Poladova⁽²⁾ and Tahir Khaniyev⁽³⁾

⁽¹⁾ *Karabük University, Department of Actuarial Sciences, Karabük, Türkiye*
e-mail: zulfiyyamammadova@gmail.com

⁽²⁾ *TOBB University of Economics and Technology, Department of Basic Medical Sciences, Ankara, Türkiye*
e-mail: aynurapoladova04@gmail.com

⁽³⁾ *TOBB University of Economics and Technology, Department of Department of Industrial Engineering, Ankara, Türkiye*
e-mail: tahirkhaniyev@gmail.com

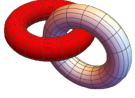
Abstract

In this study, a random walk process is considered and two important boundary functionals of the process, $N(z)$ and $S_{N(z)}$, are investigated. First, using Dynkin's principle together with the ladder times and ladder heights of the random walk process, the boundary functionals $N(z)$ and $S_{N(z)}$ are represented as renewal-reward processes. Then, we use basic methods from renewal theory to derive approximate formulas for the expected value and variances of these boundary functionals. It is demonstrated that the coefficients of the obtained formulas can be expressed in terms of the moments of the first ladder time and the ladder height of the random walk process. Finally, the numerical values of these coefficients are computed for several special cases using the Monte Carlo method, leading to the proposal of novel, practical approximate formulas for the expected values and variances of the boundary functionals.

Keywords: Random walk process, Dynkin's principle, approximate formulas.

References:

- [1] W. Feller, Introduction to Probability Theory and Its Applications. Vol. II. John Wiley & Sons, Inc., New York-London-Sydney, 1971.
- [2] B. A. Rogozin, On the distribution of the first jump. Teor. Veroyatnost. i Primenen. 9 (1964), 498–515.



Exact Solutions of the s-Wave Klein-Gordon Equation With the Squared-Cotangent Potential

Aysel Özfidan⁽¹⁾ and Aysen Durmus⁽²⁾

⁽¹⁾ *Tarsus University, Department of Natural and Mathematical Sciences, Mersin, Türkiye*
e-mail: ayselozfidan@tarsus.edu.tr

⁽²⁾ *Erciyes University, Department of Physics, Kayseri, Türkiye*
e-mail: aysend@erciyes.edu.tr

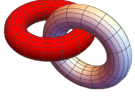
Abstract

We present exact solutions of the Klein-Gordon equation in the presence of the squared cotangent potential [1,2] within the framework of the asymptotic iteration method (AIM). Analytical solutions are derived by initially performing a coordinate transformation that reduces the Klein-Gordon equation to a second-order differential equation of a convenient form. In this context, we report the relativistic energy spectrum and radial wavefunctions for the squared-cotangent potential. These wavefunctions contain special functions that can be defined as hypergeometric functions [3]. It should be noted that AIM [4–6] is a rigorous and systematic approach to achieve quite reliable and accurate results in terms of modeling important physical systems.

Keywords: Trigonometric potential, asymptotic iteration method, hypergeometric function.

References:

- [1] M. G. Marmorino, Exactly soluble Hamiltonian with a squared cotangent potential. *J. Math. Chem.* 32 (2002), no. 3, 303–308.
- [2] D. ter Haar, *Problems in Quantum Mechanics*. 3rd Edition. Dover Publications, Inc., Mineola, New York, 2014.
- [3] I. S. Gradshteyn and I. M. Ryzhik, *Tables of Integrals, Series and Products*. Elsevier/Academic Press, Amsterdam, 2007.
- [4] H. Ciftci, R. L. Hall and N. Saad, Asymptotic iteration method for eigenvalue problems. *J. Phys. A* 36 (2003), no. 47, 11807–11816.
- [5] H. Ciftci, R. L. Hall and N. Saad, Construction of exact solutions to eigenvalue problems by the asymptotic iteration method. *J. Phys. A* 38 (2005), no. 5, 1147–1155.
- [6] H. Ciftci, R. L. Hall and N. Saad, Iterative solutions to the Dirac equation. *Phys. Rev. A* (3) 72 (2005), no. 2, part A, 022101, 7 pp.



Relativistic Treatment of Spin-0 Particles Under the Trigonometric Scarf Potential

Aysel Özfidan

Tarsus University, Department of Natural and Mathematical Sciences, Mersin, Türkiye
e-mail: ayselozfidan@tarsus.edu.tr

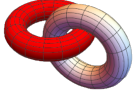
Abstract

We investigate s-state solutions of the Klein-Gordon equation with equal scalar and vector trigonometric Scarf potentials [1] using the functional analysis approach. We obtain the energy eigenvalues and the corresponding eigenfunctions in relativistic theory. It is shown that the radial wavefunctions can be expressed as hypergeometric functions and Jacobi polynomials [2]. Remarkably, in the non-relativistic limit, the present results are in excellent agreement with those obtained via the Nikiforov-Uvarov method [1].

Keywords: Jacobi polynomials, trigonometric potential, functional analysis approach.

References:

- [1] Ö. Yeşiltaş, PT/non-PT symmetric and non-Hermitian Pöschl-Teller-like solvable potentials via Nikiforov-Uvarov method. Phys. Scr. 75 (2007), no. 1, 41–46.
- [2] I. S. Gradshteyn and I. M. Ryzhik, Tables of Integrals, Series and Products. Elsevier/Academic Press, Amsterdam, 2007.



Fractional Reaction-Diffusion Modeling and Numerical Simulation of DNA Sequence Dynamics Using a Unified FD-FEM Framework

Aytekin Enver⁽¹⁾ and Fatma Ayaz⁽²⁾

⁽¹⁾*Gazi University, Department of Mathematics, Ankara, Türkiye*
e-mail: aytekinanwer@gmail.com

⁽²⁾*Gazi University, Department of Mathematics, Ankara, Türkiye*
e-mail: fayaz@gazi.edu.tr

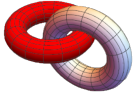
Abstract

This study introduces a novel fractional reaction-diffusion framework for analyzing DNA sequence dynamics by integrating generalized Caputo time-fractional derivatives, multi-term memory effects, and constant biological delay. Building upon the concept of fractional dynamics in DNA signal processing, the proposed model describes the temporal evolution of nucleotide interactions as a delayed multi-term time-fractional diffusion equation. A high-order finite difference method is employed for the discretization of generalized fractional operators in time, while the spatial structure of DNA strands is approximated using the finite element method. A new discrete fractional sequence enables unconditional stability analysis and establishes a delay-dependent Gronwall inequality. The framework achieves temporal convergence of order $2-\alpha$ and second-order spatial accuracy. Numerical simulations demonstrate that the proposed approach successfully captures long-range memory, fractal structure, and inter-chromosomal relationships in DNA sequences. This work provides a modern and computationally efficient bridge between fractional calculus, biological modeling, and numerical simulation for genomic analysis.

Keywords: DNA dynamics, generalized Caputo fractional derivative, multi-term time-fractional diffusion.

References:

- [1] J. A. T. Machado, A. C. Costa and M. D. Quelhas, Fractional dynamics in DNA. *Commun. Nonlinear Sci. Numer. Simul.* 16 (2011), no. 8, 2963–2969.
- [2] J. Li, J. Zhang and Y. Jiang, A finite difference method for Caputo generalized time-fractional diffusion equations. *Fractal Fract.* 10 (2026), no. 1, 19–32.



Fixed Point Methods for Nonlinear Boundary Value Problems in Complex Spaces

Bahriye Karaca

İzmir Bakırçay University, Department of Fundamental Sciences, İzmir, Türkiye
e-mail: bahriye.karaca@bakircay.edu.tr

Abstract

Boundary value problems for complex partial differential equations constitute a central topic in modern complex analysis and operator theory, with significant applications in mathematical physics and nonlinear analysis. In this study, we develop a general analytical framework for investigating boundary value problems associated with higher-order complex differential operators in complex domains. The main objective is to establish existence and uniqueness results by employing fixed point techniques in suitable functional spaces.

The proposed approach relies on transforming the original boundary value problem into an equivalent operator equation through integral representations derived from complex analytic methods. By introducing appropriate integral operators acting on complex Banach spaces, the problem is reformulated as a nonlinear operator equation whose solvability can be analyzed using fixed point theory. The structural properties of the associated operators, including boundedness, compactness, and continuity, are carefully examined to ensure the applicability of contraction-type principles.

Under general assumptions such as Lipschitz continuity and suitable growth conditions on the nonlinear terms, we prove the existence of solutions via the Banach fixed point theorem and related fixed point results. The framework is formulated in a sufficiently general setting, allowing its application to a broad class of boundary value problems without restriction to a specific boundary condition or geometry.

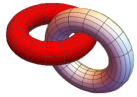
The results provide a unified functional-analytic perspective on complex boundary value problems and highlight the effectiveness of fixed point methods in complex spaces. This approach offers a flexible basis for further extensions to nonlinear higher-order equations, generalized analytic functions, and operator-theoretic formulations in complex analysis.

Keywords: Complex boundary value problems, fixed point theory, nonlinear complex equations.

Acknowledgements: The author has been supported by the Scientific and Technological Research Council of Turkey (TUBITAK) under the 2224-B Grant Program for Participation in Scientific Meetings within the Country. The author thanks TUBITAK for its support.

References:

- [1] H. Begehr, Boundary value problems for the inhomogeneous polyanalytic equation I. Analysis (Munich) 25 (2005), no. 1, 55–71.
- [2] Ü. Aksoy, H. Begehr and A. O. Çelebi, Schwarz problem for higher-order complex partial differential equations in the upper half plane. Math. Nachr. 292 (2019), no. 6, 1183–1193.



A Fibonomial Version of Truncated-Exponential-Based Appell-Type Changhee Polynomials

Barış Can Çakır⁽¹⁾ and Esra Erkus-Duman⁽²⁾

⁽¹⁾ *Gazi University, Graduate School of Natural and Applied Sciences,
Department of Mathematics, Ankara, Türkiye
e-mail: b.cancakir@yandex.com*

⁽²⁾ *Gazi University, Faculty of Science, Department of Mathematics, Ankara, Türkiye
e-mail: eduman@gazi.edu.tr*

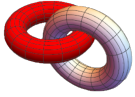
Abstract

In this study, a new family of polynomials, called the Truncated-Exponential-Based Appell-Type Changhee F-Polynomials, is introduced by combining the truncated-exponential-based Appell-type Changhee polynomial structure with Fibonomial calculus. The proposed polynomial sequence extends the corresponding classical family to the Fibonacci setting and preserves its essential algebraic characteristics. Several fundamental properties of these polynomials are established, including explicit representations, recurrence formulas, and operational identities. A determinantal representation is also obtained, providing an alternative characterization of the introduced sequence. Furthermore, the associated polynomial matrix is constructed, and its connection with the generalized Fibo-Pascal matrix is investigated. Finally, the generating function is further generalized by incorporating suitable auxiliary functions, yielding bilinear and bilateral generating-function identities together with a variety of special cases that recover several known results.

Keywords: Truncated-exponential polynomials, Changhee polynomials, Appell-type Changhee polynomials, generating function.

References:

- [1] E. Krot, An introduction to finite Fibonomial calculus. Cent. Eur. J. Math. 2 (2004), no. 5, 754–766.
- [2] J. G. Lee, L. C. Jang, J. J. Seo, S. K. Choi and H. I. Kwon, On Appell-type Changhee polynomials and numbers. Adv. Difference Equ. 2016, Paper No. 160, 10 pp.
- [3] G. Dattoli, C. Cesarano and D. Sacchetti, A note on truncated polynomials. Appl. Math. Comput. 134 (2003), no. 2-3, 595–605.
- [4] T. Nahid, P. Alam and J. Choi, Truncated-exponential-based Appell-type Changhee polynomials. Symmetry 12 (2020), no. 10, Paper No. 1588, 18 pp.



A Multinode Shepard Framework for the Numerical Solution of Integral Equations

Benaissa Zerroudi⁽¹⁾

*Ibn Zohr University, Department of Mathematics, Agadir, Morocco
e-mail: zerroudi@gmail.com*

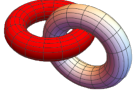
Abstract

This presentation focuses on the application of Multinode Shepard Methods (MSM) for approximating the solutions of integral equations. Unlike traditional interpolation techniques that require structured meshes, MSM provides a powerful meshless alternative capable of handling complex geometries and scattered nodal distributions. The MSM enhances the accuracy and stability of the numerical scheme. By analyzing the construction of the MSM operator, we show its versatility in transforming integral problems into manageable algebraic systems, offering a robust balance between computational cost and convergence rates.

Keywords: Integral equations, meshless methods, collocation method, multinode Shepard method.

References:

- [1] B. Zerroudi, O. Nouisser and D. Barrera, Approximate solution of Fredholm integral equations of the second kind through multinode Shepard operators. *Math. Comput. Simulation* 223 (2024), 485–493.
- [2] F. Dell’Accio, F. Di Tommaso, K. Hormann, Reconstruction of a function from hermite–birkhoff data. *Appl. Math. Comput.* 318 (2018), 51–69.
- [3] A.-M. Wazwaz, *Linear and Nonlinear Integral Equations*. Vol. 639. Higher Education Press, Beijing; Springer, Heidelberg, 2011.



Approximation Properties of Multivariate Max-Min Neural Network Operators of Durrmeyer Type

Berke Şahin⁽¹⁾ and İsmail Aslan⁽²⁾

⁽¹⁾ *Hacettepe University, Department of Mathematics, Ankara, Türkiye*
e-mail: berke.sahin@hacettepe.edu.tr

⁽²⁾ *Hacettepe University, Department of Mathematics, Ankara, Türkiye*
e-mail: ismail-aslan@hacettepe.edu.tr

Abstract

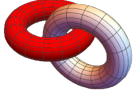
In recent years, nonlinear neural network operators have gained significant importance in approximation theory, particularly following the introduction of maximum-minimum neural network operators in [1]. Subsequent research has led to the development of their Kantorovich-type generalizations to handle broader classes of functions [2]. While linear Durrmeyer-type operators have also been investigated [3], this study introduces a novel Durrmeyer-type generalization of multivariate max-min neural network operators. The univariate version of these operators was previously addressed in [4].

We systematically investigate the approximation behavior of these operators within L^p -spaces ($1 \leq p < \infty$). Theoretical contributions include the establishment of pointwise, uniform, and L^p -convergence, alongside quantitative estimates for the rates of approximation using the modulus of continuity and K-functionals. Numerical experiments demonstrate that our integral-based Durrmeyer-type max-min operators yield superior signal denoising performance against both Gaussian and impulsive noise when compared to the operators in [1, 2]. Finally, we apply our operator to image processing tasks, such as image decompression and edge detection.

Keywords: Nonlinear operators, max-min operators, Durrmeyer-type operators, neural network operators, signal filtering, approximation theory.

References:

- [1] İ. Aslan, Approximation by max-min neural network operators. *Numer. Funct. Anal. Optim.* 46 (2025), no. 4-5, 374–393.
- [2] İ. Aslan, S. De Marchi and W. Erb, L^p -convergence of Kantorovich-type max-min neural network operators. *arXiv preprint, arXiv:2407.03329* (2024).
- [3] L. Coroianu, D. Costarelli, M. Natale and A. Pantiş, The approximation capabilities of Durrmeyer-type neural network operators. *J. Appl. Math. Comput.* 70 (2024), no. 5, 4581–4599.
- [4] B. Şahin and İ. Aslan, On the L^p -convergence and denoising performance of Durrmeyer-type max-min neural network operators. *arXiv preprint, arXiv:2601.22174* (2026).



Exponential Trapezoidal Product Integration Schemes on Graded Meshes for a Tempered Type Integrodifferential Equation

Hasret Demirci⁽¹⁾ and Betul Hicdurmaz⁽²⁾

⁽¹⁾ *Istanbul Medeniyet University, Department of Mathematics, Istanbul, Türkiye*
e-mail: hasretdemirci54@gmail.com

⁽²⁾ *Istanbul Medeniyet University, Department of Mathematics, Istanbul, Türkiye*
e-mail: betul.hicdurmaz@medeniyet.edu.tr

Abstract

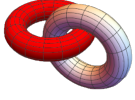
The importance of evaluating the tempered fractional integral has grown as a result of the increasing popularity of tempered fractional equations in mathematics and applied sciences [1]. In the present study, a numerical investigation of a fully discrete Crank-Nicolson scheme for a tempered-type integrodifferential equation with a diffusion operator is presented [3]. Graded temporal meshes are implemented to eliminate the Runge phenomenon [2].

The experiments confirm that the ETPI (Exponential Trapezoidal Product-Integration) method achieves stable second-order convergence for different problems. Comparisons show that other hybrid schemes yield unstable results in cases of different fractional parameters or periodic exact functions.

Keywords: Tempered fractional integral, exponential trapezoidal product integration, Crank-Nicolson scheme.

References:

- [1] H. Brunner, H. Brunner, Collocation Methods for Volterra Integral and Related Functional Equations. Cambridge University Press, Cambridge, 2004.
- [2] M. Stynes, E. O’Riordan and J. L. Gracia, Error analysis of a finite difference method on graded meshes for a time-fractional diffusion equation. SIAM J. Numer. Anal. 55 (2017), no. 2, 1057–1079.
- [3] W. Qiu, O. Nikan and Z. Avazzadeh, Numerical investigation of generalized tempered-type integrodifferential equations with respect to another function. Fract. Calc. Appl. Anal. 26 (2023), no. 6, 2580–2601.



Polynomial and Product Integration Approximations for Volterra Integral Equations with Tempered Kernels

Betul Hicdurmaz⁽¹⁾, Hacer Yildiz⁽¹⁾ and Elif Dincel⁽¹⁾

⁽¹⁾ *Istanbul Medeniyet University, Department of Mathematics, Istanbul, Türkiye*
e-mail: betul.hicdurmaz@medeniyet.edu.tr

Abstract

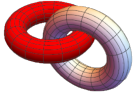
In this study, the numerical solution of second-kind Volterra integral equations whose kernels combine an algebraic weak singularity with an exponential tempering factor are studied. This structure arises naturally in anomalous diffusion and fading-memory models.

Two complementary methods are constructed and compared. The first, called Exponential-Tempered Product Integration, evaluates all singular-exponential quadrature weights exactly in closed form via the tempered incomplete gamma function. The second, Generalized Laguerre Spectral Collocation, expands the solution in a basis of weighted Laguerre polynomials that absorbs both the algebraic singularity and the exponential decay analytically. A detailed comparison in terms of error, condition number, and CPU time shows that Exponential-Tempered Product Integration method is approximately five times faster and produces a near-perfectly conditioned system, making it the more robust choice for practical computation.

Keywords: Volterra integral equation, tempered weakly singular kernel, product integration, incomplete gamma function.

References:

- [1] H. Brunner, Collocation Methods for Volterra Integral and Related Functional Equations. Cambridge University Press, Cambridge, 2004.
- [2] W. Qiu, Optimal error estimate of accurate second-order scheme for Volterra integrodifferential equations with tempered multi-term kernels. Adv. Comput. Math. 49 (2023), no. 3, Paper No. 43, 25 pp.
- [3] H. Liang and M. Stynes, A general collocation analysis for weakly singular Volterra integral equations with variable exponent. IMA J. Numer. Anal. 44 (2024), no. 5, 2725–2751.
- [4] W. Remili, A. Rahmoune and C. Li, Numerical solutions of weakly singular nonlinear Volterra integral equations with smooth and nonsmooth solutions over large time intervals. Math. Methods Appl. Sci. 48 (2025), no. 13, 12710–12724.



2D Nonstationary Wavelet Filter Design via Bézout and Sum-of-Squares Decomposition

Brahimi Mahmoud⁽¹⁾ and Laouar Mounia⁽²⁾

⁽¹⁾ *University of Batna2, Department of Mathematics, Batna, Algeria*
e-mail: m.brahimi@univ-batna2.dz

⁽²⁾ *University of Batna2, Department of Mathematics, Batna, Algeria*
e-mail: m.laoaur@univ-batna2.dz

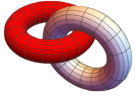
Abstract

In this work, we propose a framework for the design of two-dimensional (2D) nonstationary wavelet filter banks based on the bivariate Bézout equation in $\mathbb{R}[z_1, z_2]$, extending the approach introduced in [1]. Since the Fejér-Riesz factorization does not extend to two variables, a tensor-product sum-of-squares (TP-SOS) decomposition is used to ensure positivity, leading to a semidefinite programming (SDP) formulation. Several design criteria, including minimal support, vanishing moments, and sparsity, are considered. Numerical results show improved performance compared to separable and classical tensor-product wavelet filters.

Keywords: 2D nonstationary wavelets, wavelet filter banks, sum-of-squares (SOS).

References:

- [1] M. Brahimi, K. Melkemi and A. Boussaad, Design of nonstationary wavelets through the positive solution of Bézout's equation. *J. Interdiscip. Math.* 24 (2021), no. 3, 553–565.
- [2] I. Daubechies, Ten Lectures on Wavelets. CBMS-NSF Regional Conf. Ser. in Appl. Math., Vol. 61, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, PA, 1992.
- [3] J. B. Lasserre, Global optimization with polynomials and the problem of moments. *SIAM J. Optim.* 11 (2000/01), no. 3, 796–817.
- [4] P. A. Parrilo, Semidefinite programming relaxations for semialgebraic problems. *Math. Program.* 96 (2003), no. 2, Ser. B, 293–320.
- [5] C. Vonesch, T. Blu and M. Unser, Generalized Daubechies wavelet families. *IEEE Trans. Signal Process.* 55 (2007), no. 9, 4415–4429.



Converses of Diamond Alpha Pachpatte Type Dynamic Inequalities and Their Applications in Oscillation of Self Adjoint Linear Dynamic Equations

Bülent Koroğlu⁽¹⁾, Billur Kaymakçalan⁽²⁾ and Zeynep Kayar⁽³⁾

⁽¹⁾ *University of Turkish Aeronautical Association, Department of Common Courses, Ankara, Türkiye*
e-mail: koroglubulent@gmail.com

⁽²⁾ *University of Turkish Aeronautical Association, Department of Computer Engineering, Ankara, Türkiye*
e-mail: billurkaymakcalan@gmail.com

⁽³⁾ *Van Yüzüncü Yıl University, Department of Mathematics, Van, Türkiye*
e-mail: zeynepkayar@yyu.edu.tr

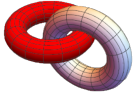
Abstract

Reverse Pachpatte-type inequalities provide concave generalizations of the established Bennett-Leindler inequalities. By leveraging concavity, we establish reverse diamond-alpha Pachpatte-type dynamic inequalities. This work marks the first instance where converses of Pachpatte-type inequalities are derived within the framework of diamond-alpha time scale calculus, encompassing its special cases: delta and nabla calculi, as well as their continuous and discrete counterparts. Moreover a novel application of reverse Pachpatte type dynamic inequalities is developed to investigate oscillation of the second order linear self adjoint dynamic equations with mixed derivatives [1].

Keywords: The diamond-alpha time scale calculus, Hardy-Copson inequality, Bennett-Leindler inequality, Pachpatte's inequality, concavity.

References:

- [1] Z. Kayar, B. Kaymakçalan and N. N. Pelen, Diamond alpha Bennett-Leindler type dynamic inequalities and their applications. Math. Methods Appl. Sci. 45 (2022), no. 5, 2797–2819.



A Study on Tempered Fractional Calculus Involving the Multivariate Mittag-Leffler Function

Cemaliye Kürt⁽¹⁾ and Mehmet Ali Özarslan⁽²⁾

⁽¹⁾ *Final International University, Department of Computer Engineering,
Girne, TRNC Mersin 10, Türkiye
e-mail: cemaliye.kurt@final.edu.tr*

⁽²⁾ *Eastern Mediterranean University, Department of Mathematics,
Gazimagusa, TRNC Mersin 10, Türkiye
e-mail: mehmetali.ozarslan@emu.edu.tr*

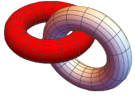
Abstract

In this paper we develop a framework for tempered fractional calculus [1] associated with the multivariate Mittag-Leffler function [2]. We start by introducing the tempered integral operator with a multivariate Mittag-Leffler kernel. Then, the fundamental properties of this operator including boundedness on $L^1(a, b)$, infinite-series representations, Laplace transform formulas, semigroup properties and the construction of a left-inverse fractional derivative operator are derived. Several examples involving power functions and generalized Mittag-Leffler functions are demonstrated.

Keywords: Multivariate Mittag-Leffler function, tempered fractional calculus, Laplace operator.

References:

- [1] F. Jarad, T. Abdeljawad and J. Alzabut, Generalized fractional derivatives generated by a class of local proportional derivatives. *Eur. Phys. J. Spec. Top.* 226 (2017), 3457–3471.
- [2] A. Abilassan, J. E. Restrepo and D. Suragan, On a variant of multivariate Mittag-Leffler's function arising in the Laplace transform method. *Integral Transforms Spec. Funct.* 34 (2023), no. 3, 244–260.
- [3] M. A. Özarslan and A. Fernandez, On the fractional calculus of multivariate Mittag-Leffler functions. *Int. J. Comput. Math.* 99 (2022), no. 2, 247–273.
- [4] R. Saxena, S. Kalla and R. Saxena, Multivariate analogue of generalized Mittag-Leffler function. *Integral Transforms Spec. Funct.* 22 (2011), no. 7, 533–548.



From Convergence Theory to Data Analysis: Is a New Approach Possible?

Ceylan Yalçın

Cankaya University, Department of Computer Sciences, Ankara, Türkiye
e-mail: cyalcin@cankaya.edu.tr

Abstract

The majority of modern data analysis techniques are influenced by distributional characteristics, such as central trends, variability, or relationships among data [5,6]. Nevertheless, seeing data as a sequence arranged in a particular order might facilitate an update of convergence theories employed in classical analysis inside the area of data analysis.

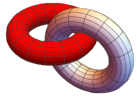
The present study suggests treating a finite dataset as a data sequence and analyzing it, particularly with respect to statistical convergence. Statistical convergence is traditionally described for infinite sequences [1–4], although the core natural density concept may also be suitably applied to limited data structures. In this context, the objective is to perform a density based evaluation of the general conduct and stability of the data sequence by analysing the ratio of observations that diverge from a fixed reference value outside a certain tolerance over the complete dataset.

The objective of this research is to examine the applicability of statistical convergence and associated generalised convergence techniques as a new mathematical tool for finite data analysis. Consequently, the objective is to convert convergence theory from a theoretical construct focused only on the asymptotic characteristics of infinite sequences into a different framework applicable to analysing the structural attributes and overarching patterns of finite datasets.

Keywords: Data analysis, convergence methods.

References:

- [1] H. Fast, Sur la convergence statistique. *Colloq. Math.* 2 (1951), 241–244.
- [2] T. Šalát, On statistically convergent sequences of real numbers. *Math. Slovaca* 30 (1980), no. 2, 139–150.
- [3] J. A. Fridy, On statistical convergence. *Analysis* 5 (1985), no. 4, 301–313.
- [4] J. S. Connor, The statistical and strong p -Cesàro convergence of sequences. *Analysis* 8 (1988), no. 1-2, 47–63.
- [5] J. W. Tukey, *Exploratory Data Analysis*. Addison-Wesley, Reading, MA, 1977.
- [6] P. J. Huber and E. M. Ronchetti, *Robust Statistics*, 2nd ed., Wiley, Hoboken, NJ, 2009.



Fixed Point Results in Generalized Rectangular Metric Spaces in the Sense of Branciari and Their Applications

Chaimaa Benzarouala

*Chouaib Doukkali University, Faculty of Sciences, El Jadida, Morocco
Laboratory of Computer Science and Mathematics and Their Applications
e-mail: Chaimaa.benzarouala94@gmail.com*

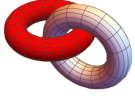
Abstract

In this work, we will introduce a new type of space called the generalized rectangular b -pseudo metric space, which generalizes the concept of metric space, rectangular metric space, b -rectangular metric space, and pseudo metric space. Next, we will prove a generalization of the famous alternative fixed point theorem of Diaz and Margolis in such space. As applications of our result, we will study the Ulam-stability of a delay differential equation with multiple variable time delays, and we will prove also the existence and uniqueness of the solution to a Fredholm integral equation. This talk is based on the article [1]

Keywords: Alternative fixed point theorem, delay differential equations, fixed point method, Fredholm integral equation, function space, gauge space, generalized metric space, multiple variable delays, rectangular b -metric space, Ulam-stability.

References:

- [1] C.Benzarouala, L.Oubbi and C.Tunç, Fixed point results in generalized rectangular metric spaces in the sense of Branciari and their applications. *Math. Methods Appl. Sci.* 49 (2026), no. 2, 668–675.
- [2] C. Benzarouala and L. Oubbi, A fixed point theorem in gauge spaces and applications to Ulam-Hyers-Rassias-stability of delay differential equations. *Afr. Mat.* 36 (2025), no. 1, Paper No. 28, 14 pp.



Strict Hölder Stability of Perturbed Nonlinear Matrix Differential Systems with Initial Time Difference

Coşkun Yakar⁽¹⁾ and Dilara Karşoğlu⁽²⁾

⁽¹⁾*Gebze Technical University, Department of Mathematics, Kocaeli, Türkiye*
e-mail: cyakar@gtu.edu.tr

⁽²⁾*Fenerbahçe University, Department of Management Information Systems, İstanbul, Türkiye*
e-mail: dilara.karslioglu@fbu.edu.tr

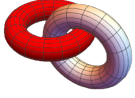
Abstract

In this paper, we investigate strict Hölder stability for perturbed nonlinear matrix differential systems with initial time difference (ITD). Using variation-of-parameters and comparison techniques, we derive two-sided Hölder-type estimates and sufficient conditions for strict Hölder stability. Our results extend existing ITD stability criteria from the Lipschitz to the Hölder setting. An example illustrates the theoretical findings.

Keywords: Perturbed matrix systems, initial time difference, strict Hölder stability.

References:

- [1] F. M. Dannan and S. Elaydi, Lipschitz stability of nonlinear systems of differential equations. *J. Math. Anal. Appl.* 113 (1986), no. 2, 562–577.
- [2] V. Lakshmikantham and A. S. Vatsala, Differential inequalities with initial time difference and applications. *J. Inequal. Appl.* 3 (1999), no. 3, 233–244.



On the Isospectrality Relation Between PT-Symmetric Schrödinger Operators and Their Adjoints

Ece Özdemir

Pamukkale University, Department of Mathematics, Denizli, Türkiye
e-mail: eceozdemir@pau.edu.tr

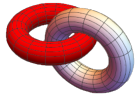
Abstract

We consider the one-dimensional Schrödinger operator $L(q)y = -y'' + q(x)y$ generated by a complex-valued, locally integrable, 1-periodic potential q satisfying the PT-symmetry condition $\overline{q(-x)} = q(x)$. Such operators are, in general, non-self-adjoint, yet they form a distinguished class whose spectra retain several features of the self-adjoint setting. This talk concerns a structural property underlying that behaviour: the adjoint $L^*(q)$ of a PT-symmetric operator $L(q)$ is again PT-symmetric. We then indicate how this property leads to the spectral equality $\sigma(L(q)) = \sigma(L^*(q))$ between the operator and its adjoint, so that the reality of the spectrum in this class can be traced back to the isospectrality between $L(q)$ and $L^*(q)$ rather than to self-adjointness.

Keywords: PT-symmetry, non-self-adjoint Schrödinger operator, adjoint operator, isospectrality.

References:

- [1] C. M. Bender and S. Boettcher, Real spectra in non-Hermitian Hamiltonians having PT symmetry. *Phys. Rev. Lett.* 80 (1998), no. 24, 5243–5246.
- [2] O. A. Veliev, Spectral analysis of the Schrödinger operator with a PT-symmetric periodic optical potential. *J. Math. Phys.* 61 (2020), no. 6, 063508, 19 pp.
- [3] O. A. Veliev, *Non-Self-Adjoint Schrödinger Operator with a Periodic Potential*. Springer, Cham, 2021.



Approximation in Weighted Spaces by Bivariate Max-Product Operators

Ecem Acar^(1,2) and Sevilay Kırıcı Serenbay^(3,4)

⁽¹⁾ *Mersin University, Department of Mathematics, Mersin, Türkiye*

⁽²⁾ *Harran University, Department of Mathematics Education, Şanlıurfa, Türkiye*
e-mail: ecemkarakus33@gmail.com

⁽³⁾ *Ankara University, Department of Mathematics, Ankara, Türkiye*

⁽⁴⁾ *Harran University, Department of Mathematics, Şanlıurfa, Türkiye*
e-mail: skserenbay@ankara.edu.tr

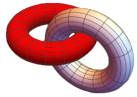
Abstract

In this paper, we introduce a new weighted setting for the study of approximation properties of bivariate max-product operators. A weighted modulus of continuity adapted to the underlying approximation process is constructed, and its fundamental properties are established. This approach is used to examine the approximation properties of bivariate max-product Favard–Szász–Mirakyan and Meyer–König–Zeller operators. Quantitative estimates for the degree of approximation are obtained in terms of the proposed weighted modulus. The developed approach enables the analysis of approximation processes on both bounded and unbounded domains within a unified framework. The obtained results extend existing approximation estimates for bivariate nonlinear max-product operators and contribute to the theory of multivariate nonlinear approximation.

Keywords: Bivariate max-product operators, weighted modulus of continuity, degree of approximation.

References:

- [1] H. N. Mhaskar, Weighted polynomial approximation. *J. Approx. Theory* 46 (1986), no. 1, 100–110.
- [2] Z. Ditzian and V. Totik, *Moduli of Smoothness*, Springer-Verlag, New York, 1987.
- [3] S. G. Gal, *Shape-Preserving Approximation by Real and Complex Polynomials*, Birkhäuser Boston, Inc., Boston, MA, 2008.
- [4] B. Bede, L. Coroianu and S. G. Gal, *Approximation and Shape Preserving Properties of the Max-Product Operators*, Springer, Heidelberg, 2016.
- [5] B. Bede, L. Coroianu and S. G. Gal, Approximation and shape preserving properties of the Bernstein operator of max-product kind. *Int. J. Math. Math. Sci.* 2009, Art. ID 590589, 26 pp.
- [6] G. A. Anastassiou and S. G. Gal, *Approximation Theory: Moduli of Continuity and Global Smoothness Preservation*, Birkhäuser, Boston, 2000.
- [7] L. Rempulska and M. Skorupka, On Szász–Mirakyan operators of functions of two variables, *Le Matematiche* 53(1998), no. 1, 51–60.
- [8] A. Holhoş, Weighted approximation of functions by Meyer–König and Zeller operators of max-product type. *Numer. Funct. Anal. Optim.* 39 (2018), no. 6, 689–703.
- [9] A. Holhoş, Approximation of functions by Favard–Szász–Mirakyan operators of max-product type in weighted spaces. *Filomat* 32 (2018), no. 7, 2567–2576.



Volterra Integral Equation Framework for Optimal Drug Administration in Tumor Growth Control

El Amir Djeffal⁽¹⁾ and Randa Chalekh⁽²⁾

⁽¹⁾ *University of Batna 2, Department of Mathematics, Batna, Algeria*
e-mail: l.djeffal@univ-batna2.dz

⁽²⁾ *University of Batna 2, Department of Mathematics, Batna, Algeria*
e-mail: r.chalekh@univ-batna2.dz

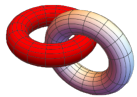
Abstract

This paper presents a mathematical framework for modeling tumor growth under treatment using Volterra integral equations. The system incorporates memory effects inherent in biological processes. An optimal control problem is formulated to determine the best drug administration strategy that minimizes tumor size while reducing drug toxicity. Numerical methods, including discretization and optimization techniques such as gradient-based and interior-point methods, are discussed. Simulation results demonstrate the effectiveness of the proposed approach.

Keywords: Volterra integral equations, optimal control, tumor growth modeling, numerical simulation, medical treatment optimization.

References:

- [1] V. Volterra, Theory of Functionals and of Integral and Integro-Differential Equations. Dover Publications, Inc., New York, 1959.
- [2] G. Gripenberg, S. O. Londen and O. Staffans, Volterra Integral and Functional Equations. Cambridge University Press, Cambridge, 1990.
- [3] J. D. Murray, Mathematical Biology I. Springer-Verlag, New York, 2002.
- [4] N. Bellomo and L. Preziosi, Modelling and mathematical problems related to tumor evolution and its interaction with the immune system. Math. Comput. Modelling 32 (2000), no. 3-4, 413–452.
- [5] S. Lenhart and J. T. Workman, Optimal Control Applied to Biological Models. Chapman & Hall/CRC, Boca Raton, FL, 2007.
- [6] L. C. Evans, An Introduction to Mathematical Optimal Control Theory. UC Berkeley, 2005.
- [7] S. Boyd and L. Vandenberghe, Convex Optimization. Cambridge University Press, Cambridge, 2004.
- [8] M. Raissi, P. Perdikaris, and G. E. Karniadakis, Physics-informed neural networks: A deep learning framework for solving forward and inverse problems involving nonlinear partial differential equations. J. Comput. Phys. 378 (2019), 686–707.



Enhancing Feature Selection in Molecular Dynamics Trajectory Data for Hidden Markov Models

Elif Naz Fidancı⁽¹⁾ and Vilda Purutçuoğlu^(1,2)

⁽¹⁾ *Middle East Technical University, Department of Biomedical Engineering, Ankara, Türkiye*
e-mail: elif.fidanci@metu.edu.tr

⁽²⁾ *Middle East Technical University, Department of Statistics, Ankara, Türkiye*
e-mail: vpurutcu@metu.edu.tr

Abstract

Hidden Markov Models (HMMs) are well-established probabilistic models that have been successfully used in genomic and protein analyses to identify meaningful sequence patterns, functional regions, and binding sites associated with molecular recognition. However, their integration into molecular dynamics (MD) simulation analysis, particularly with systematic feature selection, represents a relatively novel application. This study integrates HMMs with MD trajectory data to identify hidden kinetic states and conformational transitions.

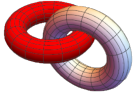
Although HMMs are effective for analysing complex, high-dimensional time-series data, their accuracy and interpretability may be limited by the initial selection of input features. Conventionally, all available features may be included without a specific feature-selection procedure. However, irrelevant, redundant, or highly correlated features can introduce noise and obscure physically meaningful molecular states. Therefore, this study aims to identify and retain only the most informative features before constructing the HMM.

A feature-selection pipeline combining LightGBM with SHAP is proposed. LightGBM models the relationships within the trajectory data, while SHAP evaluates the relative importance of individual features. The VAMP score is also employed as a complementary, kinetically informed feature-selection method. The selected features are subsequently used to construct the HMM.

The framework is evaluated using MD trajectory data from the WLALL pentapeptide, consisting of tryptophan, leucine, alanine, leucine, and leucine residues. Its performance is also compared with Time-Lagged Independent Component Analysis (TICA), a conventional dimensionality-reduction method.

The results show that the proposed framework is promising for MD trajectory analysis. The LightGBM-SHAP and VAMP approaches successfully identify important features while reducing redundant and noise-generating variables. This enables the HMM to detect physically meaningful kinetic states more clearly. Overall, integrating feature selection with HMM analysis provides a more interpretable and effective approach for understanding hidden conformational states and molecular transitions in MD simulation data.

Keywords: Molecular dynamics simulations, hidden Markov model, explainable feature selection.



Fluctuationlessness Theorem Applied to Caputo Fractional Initial Value Problems

Ercan Gürvit

Marmara University, Faculty of Sciences, Department of Mathematics, İstanbul, Türkiye
e-mail: ercangurvit@gmail.com

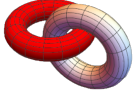
Abstract

We adapt the Fluctuationlessness Theorem [1]—which represents a scalar function of the independent-variable operator, on a finite-dimensional subspace, by that function evaluated on the matrix of the independent variable once fluctuation terms are neglected—to the numerical solution of Caputo fractional initial value problems. Rewriting such a problem as a weakly singular Volterra integral equation, we discretize the memory integral panel by panel: on each step the integral splits into regular history panels and a single weakly singular diagonal panel. The diagonal panel collapses exactly, and independently of the mesh, onto a normalized Jacobi weight $w_\alpha(\sigma) = \alpha(1 - \sigma)^{\alpha-1}$, for which the theorem supplies a quadrature—computed once and stored—whose nodes accumulate at the endpoint singularity. Predictor-corrector marching schemes are built on this splitting [2]. Because the diagonal panel is treated exactly and mesh-independently, the method accommodates graded meshes that cluster points near the initial singularity, avoiding the accuracy loss suffered by closed-form product weights [3]. A detailed error and convergence analysis on graded meshes is developed in a separate, fuller study. Linear, nonlinear, and weakly singular numerical tests illustrate the accuracy of the approach.

Keywords: Fluctuationlessness theorem, Caputo fractional derivative, product integration, weakly singular Volterra equation, Jacobi weight, graded mesh, predictor-corrector.

References:

- [1] M. Demiralp, Fluctuationlessness theorem to approximate univariate functions' matrix representations. WSEAS Trans. Math. 8 (2009), no. 6, 258–267.
- [2] K. Diethelm, N. J. Ford and A. D. Freed, A predictor-corrector approach for the numerical solution of fractional differential equations. Nonlinear Dynam. 29 (2002), no. 1-4, 3–22.
- [3] H. Brunner, Collocation Methods for Volterra Integral and Related Functional Differential Equations. Cambridge Monogr. Appl. Comput. Math., Vol. 15, Cambridge University Press, Cambridge, 2004.



Fourier Transforms of Orthogonal Polynomials on The Double Cone

Esra Çekirdek⁽¹⁾ and Rabia Aktaş Karaman⁽²⁾

⁽¹⁾ *Ankara University, Graduate School of Natural and Applied Sciences, Ankara, Türkiye*
e-mail: ecekirdek@ankara.edu.tr

⁽²⁾ *Ankara University, Department of Mathematics, Ankara, Türkiye*
e-mail: raktas@science.ankara.edu.tr

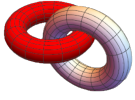
Abstract

The purpose of this paper is to obtain Fourier transforms of multivariate orthogonal polynomials on the double cone such as generalized Gegenbauer polynomials on the double cone and generalized Hermite polynomials on the double cone, and to define two new families of multivariate orthogonal functions by using Parseval's identity. In addition, some properties of these functions will be investigated.

Keywords: Generalized Gegenbauer polynomials, generalized Hermite polynomials, multivariate orthogonal polynomials, Fourier transform, Parseval's identity.

References:

- [1] R. Aktaş Karaman and I. Area, Fourier transforms of orthogonal polynomials on the cone, Numer. Algor. 2025 (*in press*).
- [2] H. Ö. Çetin and R. Aktaş Karaman, Fourier transforms of orthogonal structures on the paraboloid. Mediterr. J. Math. 23 (2026), no. 3, Paper No. 119.
- [3] E. Göldoğan Lekesiz, R. Aktaş and I. Area, Fourier transform of orthogonal polynomials on the unit ball and continuous Hahn polynomials. Axioms 11 (2022), no. 10, 558, 15 pp.
- [4] H. T. Koelink, On Jacobi and continuous Hahn polynomials. Proc. Amer. Math. Soc. 124 (1996), no. 3, 887–898.
- [5] M. Masjed Jamei and W. Koepf, Two classes of special functions using Fourier transforms of generalized ultraspherical and generalized Hermite polynomials. Proc. Amer. Math. Soc. 140 (2012), no. 6, 2053–2063.
- [6] Orthogonal structure and orthogonal series in and on a double cone or a hyperboloid. Trans. Amer. Math. Soc. 374 (2021), no. 5, 3603–3657.



A Novel Set of Univariate Biorthogonal Polynomials

Esra Gldođan Lekesiz

Çankaya University, Department of Mathematics, Ankara, Türkiye
e-mail: esraguldoganlekesiz@cankaya.edu.tr

Abstract

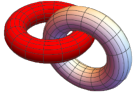
As is known, biorthogonal polynomials are an extension of orthogonal polynomials and, for some special values, can be reduced to a scalar case. Constructing a biorthogonal structure from scratch is quite difficult because biorthogonality is related to the orthogonality of two different families. So far, four families of one-variable biorthogonal polynomials have been defined in the literature.

In this study, a new class of biorthogonal polynomial families has been defined. In addition to many properties obtained for this family, studies have also been conducted on fractional operators and integral transformations for these polynomials.

Keywords: Biorthogonal polynomial, finite orthogonal polynomial, integral transform.

References:

- [1] H. C. Madhekar and N. K. Thakare, Biorthogonal polynomials suggested by the Jacobi polynomials. *Pacific J. Math.* 100 (1982), no. 2, 417–424.
- [2] M. Masjed-jamei, Three finite classes of hypergeometric orthogonal polynomials and their application in functions approximation. *Integral Transforms Spec. Funct.* 13 (2002), no. 2, 169–191.
- [3] E. Gldođan Lekesiz, Finite biorthogonal polynomials suggested by the finite orthogonal polynomials $M_n^{(p,q)}(x)$. *Math. Methods Appl. Sci.* 49 (2026), no. 11, 12524–12538.



Operational Properties of Two-Parameter Laguerre-Type Sheffer Polynomials

Esra Karaoğlu⁽¹⁾, Ceylan Yalçın⁽²⁾ and Esra Erkuş-Duman⁽³⁾

⁽¹⁾ *Çankaya University, Department of Mathematics, Ankara, Türkiye*
e-mail: ekaraoglu@cankaya.edu.tr

⁽²⁾ *Çankaya University, Department of Computer Science, Ankara, Türkiye*
e-mail: cyalcin@cankaya.edu.tr

⁽³⁾ *Gazi University, Department of Mathematics, Ankara, Türkiye*
e-mail: eduman@gazi.edu.tr

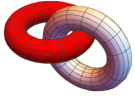
Abstract

Sheffer polynomial sequences have been extensively studied due to their rich algebraic structure and numerous applications in applied mathematics, approximation theory, combinatorics, and mathematical physics [1–3]. Since both Appell and associated polynomial sequences belong to the Sheffer class, many well-known polynomial families can be treated within a common algebraic framework. Motivated by these works, we introduce a new family of two-parameter polynomials and study their generating function, explicit representations, recurrence relations, differential identities, and operational formulas. We also establish generating function identities and investigate the influence of the parameters on the structure of the polynomials. Furthermore, connections with several well-known polynomial families are discussed by considering appropriate parameter choices, showing that the proposed sequence provides a unified framework for a variety of classical polynomials. Illustrative examples of the first few polynomials are presented to demonstrate the theoretical results.

Keywords: Sheffer sequences, Laguerre polynomials, generating function.

References:

- [1] S. Roman, *The Umbral Calculus*, Academic Press, New York, 1984.
- [2] F. A. Costabile, M. I. Gualtieri, and A. Napoli, Towards the centenary of Sheffer polynomial sequences: old and recent results. *Mathematics*. 10 (2022), no. 23, 4435, 28 pp.
- [3] S. Khan and N. Raza, Monomiality principle, operational methods and family of Laguerre–Sheffer polynomials. *J. Math. Anal. Appl.* 387 (2012), no. 1, 90–102.



k-Hypergeometric Functions and Applications

Ezgi Alkan^(1,2) and Durmuş Albayrak⁽²⁾

⁽¹⁾*Haliç University, Department of Mathematics, İstanbul, Türkiye*
e-mail: ezgialkan@halic.edu.tr

⁽²⁾*Marmara University, Department of Mathematics, İstanbul, Türkiye*
e-mail: durmus.albayrak@marmara.edu.tr

Abstract

Hypergeometric functions play a central role in the theory of special functions, as they provide a common framework for studying the solutions of many differential equations [3]. With suitable choices of parameters, they reduce to several well-known functions such as Bessel and Legendre functions. A natural extension of these classical structures is given by the k-hypergeometric functions, which are defined in terms of the k-Gamma function and the k-Pochhammer symbol, and were introduced by Diaz and Pariguan [1].

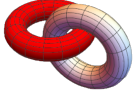
In this work, we first recall some basic properties of classical hypergeometric series and then examine how these structures can be generalized by means of the k-parameter. The main focus is on the k-extensions of balanced and well-balanced hypergeometric series. In this context, k-analogues of several classical identities and summation formulas—such as those of Saalschütz, Watson, Dixon, Whipple, and Bailey—are also considered. In particular, their convergence conditions and certain fundamental properties are discussed.

The results indicate that these k-generalized structures can be regarded as a natural continuation of classical hypergeometric functions and that they recover several known identities in the literature. From this perspective, the study highlights the role of parametric generalizations in the development of hypergeometric function theory [1, 4].

Keywords: Hypergeometric identities, generalized k-hypergeometric functions, k-pochhammer symbol.

References:

- [1] E. D. Rainville, *Special Functions*. Macmillan Company, New York, 1960; reprinted by Chelsea Publishing Company, Bronx, New York, 1971.
- [2] R. Díaz and E. Pariguan, On hypergeometric functions and Pochhammer k-symbol. *Divulg. Mat.* 15 (2007), no. 2, 179–192.
- [3] G. E. Andrews, R. Askey and R. Roy, *Special Functions*. Cambridge University Press, 1999
- [4] L. J. Slater, *Confluent Hypergeometric Functions*. Cambridge University Press, Cambridge, 1960.



Asymptotic Behavior of Sequences in Vector Lattices

Nazife Erkuşun-Özcan⁽¹⁾, Ezgi Han Eryüksel⁽²⁾ and Niyazi Anıl Gezer⁽³⁾

⁽¹⁾ *Hacettepe University, Department of Mathematics, Ankara, Türkiye*
e-mail: erkursun.ozcan@hacettepe.edu.tr

⁽²⁾ *Ankara Medipol University, Department of Computer Engineering, Ankara, Türkiye*
e-mail: ezgi.eryuksel@ankaramedipol.edu.tr

⁽³⁾ *TED University, Department of Mathematics, Ankara, Türkiye*
e-mail: anilgezer@gmail.com

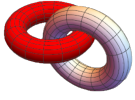
Abstract

Classical asymptotic notations O , Ω , and Θ are widely used to describe the growth behavior of sequences in functional and theoretical numerical analysis. Motivated by these classical concepts, the asymptotic relations O_a^b , Ω_a^b , and Θ_a^b for sequential convergence were introduced in [2]. In this talk, we investigate the asymptotic relations O_a^b , Ω_a^b , and Θ_a^b associated with several types of convergences defined on Banach lattices. We present various order theoretical properties of these relations, including domination and subsequence results for different convergences. We further study these asymptotic relations on regular sublattices of vector lattices. The results presented in this talk are based on the paper [1].

Keywords: Asymptotic behavior, convergence, vector lattice.

References:

- [1] N. Erkuşun-Özcan, E. H. Eryüksel and N. A. Gezer, Asymptotic analysis of demi-continuous operators with applications. 2026 (*under review*).
- [2] N. A. Gezer, Unbounded growth of sequences in vector lattices. *Positivity* 27 (2023), no. 2, Paper No. 25, 13 pp.
- [3] D. E. Knuth, *The Art of Computer Programming. Vol. 1: Fundamental Algorithms.* 3rd Edition, Addison-Wesley Publishing, Boston, 1997.



On the Pseudospectra of q -Quasinormal Operators

Meltem Serbaşı⁽¹⁾ and Fatih Yılmaz⁽²⁾

⁽¹⁾*Karadeniz Technical University, Faculty of Science, Department of Mathematics, Trabzon, Türkiye*
e-mail: m.sertbas@ktu.edu.tr

⁽²⁾*Karadeniz Technical University, Faculty of Science, Department of Mathematics, Trabzon, Türkiye*
e-mail: fatih.yilmaz@ktu.edu.tr

Abstract

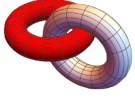
This study investigates the ε -pseudospectra of bounded q -quasinormal operators acting on separable Hilbert spaces, where $q > 1$. By using the structural representation of such operators as weighted shift-type tensor product operators, several geometric properties of their pseudospectra are obtained.

It is also shown that, as $q \rightarrow \infty$, the pseudospectra of these operators approach the pseudospectrum of a rank-one nilpotent operator. This shows that the limiting behavior is determined mainly by the nonnormal structure of the operator rather than by its spectrum.

Keywords: q -quasinormal operator, pseudospectrum.

References:

- [1] J. Cui, C. Li and Y. Poon, Pseudospectra of special operators and pseudospectrum preservers. *J. Math. Anal. Appl.* 419 (2014), no. 2, 1261–1273.
- [2] J. B. Conway, *The Theory of Subnormal Operators*. Mathematical Surveys and Monographs, Vol. 36, American Mathematical Society, Providence, RI, 1985.
- [3] L. N. Trefethen and M. Embree, *Spectra and Pseudospectra: The Behavior of Nonnormal Matrices and Operators*, Princeton University Press, Princeton, NJ, 2005.
- [4] M. Embree and L. N. Trefethen, Generalizing eigenvalue theorems to pseudospectra theorems. *SIAM J. Sci. Comput.* 23 (2001), no. 2, 583–590.
- [5] F. Yılmaz, On the connection between $\sigma_\varepsilon(A_1 \otimes A_2)$ and $\sigma_\varepsilon(A_1)$, $\sigma_\varepsilon(A_2)$ for certain special operators. *Turkish J. Math.* 48 (2024), no. 5, Art. 14, 995–1000.
- [6] B. Jia and Y. Feng, An observation about pseudospectra. *Filomat* 35 (2021), no. 3, 995–1000.



A Time-Filtered VMS-POD Reduced-Order Model for Double-Diffusive Darcy-Brinkman Equations

Fatma Güler Eroglu

*Atılım University, Department of Mathematics, Ankara, Türkiye
e-mail: fatma.guler@atilim.edu.tr*

Abstract

Double-diffusive Darcy-Brinkman equations are driven by two scalar fields with different diffusion rates. To model porous medium effects, a Darcy term is included in the momentum equation. Since the physical model consists of a coupled system of momentum, heat, and mass transfer equations, its numerical simulation is computationally demanding. Additionally, full-order numerical methods often lead to large algebraic systems and a high computational burden. Thus, efficient reduced-order techniques are required to significantly reduce the computational cost while maintaining numerical accuracy and robustness.

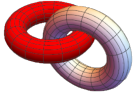
Proper Orthogonal Decomposition (POD)-based reduced-order models (ROMs) have become a popular approach for reducing the computational complexity of such problems. POD constructs a low-dimensional approximation that preserves the dominant dynamics of the original system [1, 2]. Another challenge arises as the Rayleigh number increases, causing convection cells to form and the flow to become more turbulent. To address this issue, a projection-based Variational Multiscale (VMS) method will be employed to improve numerical stability while preserving the accuracy of the reduced-order model [1].

We extend the method proposed in [3] to solve the double-diffusive Darcy-Brinkman equations. The resulting framework combines POD, projection-based VMS stabilization, and a time-filtering technique. The stability and error analyses of this method are presented, and its performance is demonstrated through benchmark test problems. Numerical results indicate that this approach improves temporal accuracy and energy preservation without increasing process time. The proposed method provides a robust, stable, and accurate reduced-order approach for the simulation of double-diffusive Darcy-Brinkman flows.

Keywords: Proper orthogonal decomposition, variational multiscale method, time filtering.

References:

- [1] F. Eroglu, S. Kaya and L. Rebholz, A modular regularized variational multiscale proper orthogonal decomposition for incompressible flows. *Comput. Methods Appl. Mech. Engrg.* 325 (2017), 350–368.
- [2] F. Eroglu, S. Kaya and L. Rebholz, POD-ROM for the Darcy-Brinkman equations with double-diffusive convection. *J. Numer. Math.* 27 (2019), no. 3, 123–139.
- [3] F. Eroglu, An adaptive time filtered backward Euler method for reduced-order models of incompressible flows. *Gazi Univ. J. Sci.* 36 (2023), no. 4, 1658–1673.



From Infinite Arrays to Finite Matrices: The Truncated Riordan Group Approach

Fatma Yeşil Baran⁽¹⁾ and Naim Tuğlu⁽²⁾

⁽¹⁾ *Amasya University, Department of Computer Engineering, Amasya, Türkiye*
e-mail: fatma.yesilbaran@amasya.edu.tr

⁽²⁾ *Gazi University, Department of Mathematics, Ankara, Türkiye*
e-mail: naimtuglu@gazi.edu.tr

Abstract

The $(n + 1)$ -th partial sum of the Maclaurin series expansion of e^x is represented by the truncated exponential polynomials [1], which are expressed as follows:

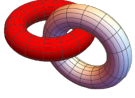
$$e_n(x) = \sum_{k=0}^n \frac{x^k}{k!}.$$

The structural function of the truncated Riordan group as a strong algebraic framework for the analysis of specific polynomial sequences will be investigated in this study. The formal definition of a truncated polynomial, which offers the required mathematical basis to limit infinite sequences into finite structures, is essential to this investigation. By confirming basic algebraic axioms under a particular operation, we will show how this algebraic step enables a formal transition from infinite-dimensional spaces to manageable finite-dimensional systems.

Keywords: Riordan array, truncated polynomial, double Riordan array.

References:

- [1] L. C. Andrews, Special functions for engineers and applied mathematicians. Macmillan Co., New York, 1985.
- [2] G. Dattoli, C. Cesarano and D. Sacchetti, A note on truncated polynomials. Appl. Math. Comput. 134 (2003), no. 2-3, 595–605.
- [3] Y. Alp, F. Y. Baran, E. G. Kocer and N. Tuglu, q -almost Riordan arrays. Filomat, 40 (2026), no. 8, 3043–3059.
- [4] S. Khan, G. Yasmin and N. Ahmad, A note on truncated exponential-based Appell polynomials. Bull. Malays. Math. Sci. Soc. 40 (2017), no. 1, 373–388.
- [5] Z. Özat, B. Çekim, M. A. Özarslan and F. A. Costabile, Truncated-exponential-based general-Appell polynomials. Mathematics, 13 (2025) no. 8, Article No. 1266, 15 pp.
- [6] L. W. Shapiro, S. Getu, W. J. Woan and L. C. Woodson, The Riordan group. Discrete Appl. Math. 34 (1991), no. 1-3, 229–239.



Smoothed Rank-Based Regression Estimation Using Wilcoxon Score Functions

Feridun Tasdan

Western Illinois University, Department of Mathematics, Macomb, USA
e-mail: f-tasdan@wiu.edu

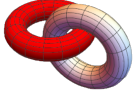
Abstract

This article proposes an improved rank-based regression estimator obtained by replacing the ordinary integer ranks in the Wilcoxon rank-score regression procedure with *smoothed ranks* derived from a smoothed empirical cumulative distribution function (ecdf). The smoothed ranks are computed via a continuous, non-decreasing kernel distribution function $H(\cdot)$ that provides a differentiable approximation to the classical indicator function used in standard rank regression. Substituting these smoothed ranks into the Wilcoxon score function yields a new estimator, denoted $\hat{\beta}_{sr}$, for the slope parameter(s) of the simple and multiple linear regression model. We show that the proposed estimator inherits the robustness properties of classical rank regression while providing improved efficiency under heavy-tailed error distributions and better handling of tied observations. A Wald-type hypothesis test for the regression coefficients is derived and its asymptotic normality is established. A Monte Carlo simulation study compares $\hat{\beta}_{sr}$ with the ordinary least-squares (OLS) estimator, the classical Wilcoxon rank regression estimator, and the Theil-Sen estimator under several error distributions including the normal, Laplace, Cauchy, and contaminated normal. The proposed estimator achieves relative efficiencies at or above those of classical rank regression uniformly across all scenarios considered, with notable gains in the presence of outliers and heavy-tailed errors.

Keywords: Rank Regression, smoothed ranks, Wilcoxon score function, robust estimation, relative efficiency, Monte Carlo simulation.

References:

- [1] L. Lin and H. Peng, Smoothed rank correlation of the linear transformation regression model. *Comput. Statist. Data Anal.* 57 (2013), 615–630.
- [2] J. W. McKean, Robust analysis of linear models. *Statist. Sci.* 19 (2004), no. 4, 562–570.
- [3] F. Tasdan, Smoothed ranks for two or multi-sample location problems. *Comm. Statist. Simulation Comput.* 47 (2018), no. 2, 526–541.
- [4] F. Tasdan and R. Dağalp, Enhanced rank-based correlation estimation using smoothed Wilcoxon rank scores. In: *Current Approaches in Applied Statistics I*, pp. 119–138, Özgür Yayınları, 2025.



A Maximal Ergodic Lemma for q -Weighted Ergodic Averages

Gencay Oğuz

Baskent University, Department of Biomedical Engineering, Ankara, Türkiye
e-mail: gencayoguz@baskent.edu.tr

Abstract

Let $(X, \mathcal{B}, \mu)$ be a probability space and let $T : X \rightarrow X$ be a measure-preserving transformation. For $f \in L^1(\mu)$, the classical ergodic averages are defined by

$$M_N f(x) := \frac{1}{N} \sum_{n=0}^{N-1} f(T^n x).$$

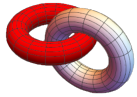
The Birkhoff pointwise ergodic theorem asserts that these averages converge almost everywhere to $\mathbb{E}(f | \mathcal{I})(x)$, where $\mathcal{I}$ denotes the T -invariant σ -algebra. A classical proof of Birkhoff's pointwise ergodic theorem is based on the maximal ergodic lemma. Several proofs of this lemma have appeared in the literature, including those of Garsia and Riesz. In parallel with these developments, various weighted versions of the maximal ergodic lemma have also been established.

In this talk, we introduce q -weighted ergodic averages and establish a q -weighted version of the classical maximal ergodic lemma for $0 < q \leq 1$ by adapting Garsia's method to the q -setting. Our proof follows the main ideas of the classical argument while making use of the structure of the associated q -weighted sums. This work provides a first step toward the study of q -weighted ergodic theory.

Keywords: Maximal ergodic lemma, ergodic averages, q -calculus.

References:

- [1] G. D. Birkhoff, Proof of the ergodic theorem. Proc. Natl. Acad. Sci. 17 (1931), no. 12, 656–660.
- [2] A. M. Garsia, A simple proof of E. Hopf's maximal ergodic theorem. J. Math. Mech. 14 (1965), no. 3, 381–382.
- [3] F. Riesz, Sur la théorie ergodique. Comment. Math. Helv. 17 (1944), 221–239.



Dunford-Pettis vs. Eberlein-Šmulian Theorems

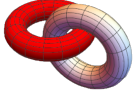
Gheorghe Bucur

University of Bucharest, Institute of Mathematics, București, Romania

Abstract

In this frame of a general duality i.e. an arbitrary function D defined on a product $X \times Y$ with values in a metric space or more generally an uniform space, we define the uniform respectively the weak topology on X and Y and we prove how the compactness property of X and Y are equivalent.

Particularly we refer to the famous result of Dunford-Pettis or Eberlein-Šmulian.



Optimizing Graph Path Encoding with Probabilistic Cryptography

Gökçe Çaylak Kayaturan

*Ordu University, Department of Mathematics, Ordu, Türkiye
e-mail: gokcekayaturan@odu.edu.tr*

Abstract

As network architectures grow in complexity, the need for efficient and private path verification becomes paramount. This paper proposes a novel framework for encoding graph paths using binary arrays integrated with ElGamal encryption to ensure both space efficiency and cryptographic security. By representing a sequence of edges or nodes as elements within a probabilistic filter, we achieve constant-time membership queries while significantly reducing the overhead associated with traditional path logging.

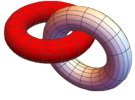
To address privacy concerns and prevent unauthorized path reconstruction, we utilize the homomorphic properties of the ElGamal cryptosystem. In this model, the bit array is constructed within the encrypted domain, or the indices are generated via ElGamal-signed identifiers. This ensures that an intermediate observer cannot determine the specific edges of a path without the corresponding private key, effectively mitigating traffic analysis attacks.

In the literature, to improve privacy protection and secure communications between parties, Diffie-Hellman key exchange protocol has been used [1]. Alternatively, we believe that the ElGamal encryption system also works perfectly with binary set representation methods for the same purpose for large-scale graph topologies. This research contributes a scalable solution for secure source routing and verifiable telemetry in decentralized networks.

Keywords: Network, ElGamal Encryption, Diffie-Hellman key exchange, graph.

References:

- [1] N. Ahmed, R. A Michelin, W. Xue, G. D. Putra, S. Ruj, S. S Kanhere and S. Jha, DIMY: Enabling privacy-preserving contact tracing. J. Network Computer Appl. 202 (2022), Article No. 103356, 14 pp.



On the Spectral and Scattering Properties of Generalized Eigenparameter-Dependent Impulsive Quantum Sturm–Liouville Equation

Güher Gülçehre Özbey⁽¹⁾ and Yelda Aygar⁽²⁾

⁽¹⁾ *Middle East Technical University, Ankara, Türkiye*
e-mail: gulcehre@metu.edu.tr

⁽²⁾ *Ankara University, Faculty of Science, Department of Mathematics, Tandoğan, Ankara, Türkiye*
e-mail: yaygar@ankara.edu.tr

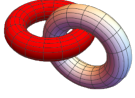
Abstract

We present an investigation about scattering analysis of boundary value problem for a q -Sturm-Liouville equation with impulsive condition and boundary conditions depending on generalized eigenvalue parameter. Here, we give Jost solution and the scattering function of this problem and we find the properties of scattering function by using the scattering solutions. We also give an asymptotic equation for the Jost solution of this problem. We also find the Green function and resolvent operator of this boundary value problem. Finally, we apply the new results on an example.

Keywords: Scattering theory, scattering function, impulsive condition.

References:

- [1] G. G. Ozbey, G. B. Oznur and Y. Aygar, Some spectral and scattering properties of generalized eigenparameter dependent discrete transmission Sturm-Liouville equation. *Honam Math. J.* 45 (2023) (2023), no. 3, 457–470.
- [2] T. Koprubasi and R. N. Mohapatra, Spectral properties of generalized eigenparameter dependent discrete Sturm-Liouville type equation. *Quaest. Math.* 40 (2017), no. 4, 491–505.
- [3] A. M. Samoilenko and N. A. Perestyuk, *Impulsive Differential Equations*. World Scientific Series on Nonlinear Science, Series A: Monographs and Treatises, World Scientific Publishing Co., Inc., River Edge, NJ, 1995.
- [4] V. A. Marchenko, *Sturm-Liouville Operators and Applications*. Operator Theory: Advances and Applications, Birkhäuser, Basel, Translated from the Russian by A. Iacob., 1986.



On Riemann-Liouville Type Fractional Generalized Kantorovich Operators

Gülen Başcanbaz-Tunca⁽¹⁾ and Ayşegül Erençin⁽²⁾

⁽¹⁾ *Ankara University, Department of Mathematics, Ankara, Türkiye*
e-mail: tunca@science.ankara.edu.tr

⁽²⁾ *Bolu Abant İzzet Baysal University, Department of Mathematics, Bolu, Türkiye*
e-mail: erencina@hotmail.com

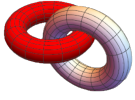
Abstract

In this talk, we introduce a modification of univariate generalized Kantorovich operators constructed in [1] via Riemann-Liouville fractional integral. We first establish the uniform convergence of these newly defined operators and provide a quantitative estimate for the rate of convergence using the first-order Ditzian-Totik modulus of smoothness. Furthermore, we investigate a Voronovskaya type theorem and simultaneous approximation.

Keywords: Kantorovich operators, Riemann-Liouville fractional integral, Ditzian-Totik modulus of smoothness, Voronovskaya type theorem, simultaneous approximation.

References:

- [1] F. Altomare, M. Cappelletti Montano, V. Leonessa and I. Raşa, A generalization of Kantorovich operators for convex compact subsets. *Banach J. Math. Anal.* 11 (2017), no. 3, 591–614.



Spectral Properties of a Discrete Impulsive Schrödinger Equation

Güler Başak Öznur⁽¹⁾ and Yelda Aygar⁽²⁾

⁽¹⁾*Gazi University, Department of Mathematics, Ankara, Türkiye*
e-mail: basakoznur@gazi.edu.tr

⁽²⁾*Ankara University, Department of Mathematics, Ankara, Türkiye*
e-mail: yaygar@ankara.edu.tr

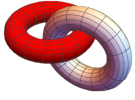
Abstract

In this work, we investigate the spectral analysis of a difference quadratic pencil of Schrödinger equation on the half line with an matrix impulsive condition. We first determine Jost solutions of the problem and present some asymptotic equations for these solutions. Then, by obtaining the resolvent operator and Green function, we find the sets of eigenvalues and spectral singularities of the impulsive problem. Furthermore, we present suitable conditions for the finiteness and multiplicities of eigenvalues and spectral singularities of the problem.

Keywords: Quadratic pencil of Schrödinger equations, impulsive condition, spectral singularities, eigenvalues, resolvent operator.

References:

- [1] M. A. Naimark, Investigation of the spectrum and the expansion in eigenfunctions of a non-selfadjoint differential operator of the second order on a semi-axis. Amer. Math. Soc. Transl. (2) 16 (1960), 103–193.
- [2] E. Bairamov, Ö. Çakar and A. M. Krall, An eigenfunction expansion for a quadratic pencil of a Schrödinger operator with spectral singularities. J. Differential Equations 151 (1999), no. 2, 268–289.
- [3] V. A. Marchenko, Sturm-Liouville Operators and Applications. Operator Theory: Advances and Applications. Birkhäuser, Basel (Translated from the Russian by A. Iacob.), 1986.
- [4] B. M. Levitan and I. S. Sargsjan, Sturm-Liouville and Dirac Operators. Kluwer Academic Publisher Group, Dordrecht, 1991.
- [5] E. Bairamov, Ö. Çakar and A. O. Çelebi, Quadratic pencil of Schrödinger operators with spectral singularities: Discrete spectrum and principal functions. J. Math. Anal. Appl. 216 (1997), no. 1, 303–320.
- [6] A. M. Samoilenko and N. A. Perestyuk, Impulsive Differential Equations. World Scientific Series on Nonlinear Science, Series A: Monographs and Treatises, World Scientific Publishing Co., Inc., River Edge, NJ, 1995.
- [7] E. Bairamov, S. Cebesoy and I. Erdal, Properties of eigenvalues and spectral singularities for impulsive quadratic pencil of difference operators. J. Appl. Anal. Comput. 9 (2019), no. 4, 1454–1469.



Improvement of Berezin Radius Inequalities on Reproducing Kernel Hilbert Spaces by Applying Cartesian Comparability

Hamdullah Başaran⁽¹⁾ and Mehmet Gürdal⁽²⁾

⁽¹⁾ *Antalya, Türkiye*
e-mail: 07hamdullahbasaran@gmail.com

⁽²⁾ *Süleyman Demirel University, Department of Mathematics, Isparta, Türkiye*
e-mail: gurdalmehmet@sdu.edu.tr

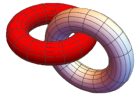
Abstract

Utilizing the Cartesian decomposition $T = T_1 + iT_2$, we define a new family of modified Berezin radius inequalities for restricted operators on reproducing kernel Hilbert spaces. Our approach depends on a comparability condition between the positive operators $|T|$ and $|T^*|$, in opposition to the conventional method based on the pair $|T_1|$ and $|T_2|$. This clearly refines the standard Cartesian-decomposition estimate for $\text{ber}^\eta(T)$ in the range $1 \leq \eta \leq 2$. Also, we present an explicit endpoint formula for the improvement constant, establish a convex-function version of the main result, and also show that this constant is the best. The sharpness and gain of the new boundaries are shown by a number of instances on a discrete reproducing kernel Hilbert space.

Keywords: Berezin number, reproducing kernel Hilbert space, Cartesian decomposition, operator inequalities, convexity.

References:

- [1] N. Aronzaajn, Theory of reproducing kernels. Trans. Amer. Math. Soc. 68 (1950), 337–404.
- [2] H. Başaran, M. Gürdal and A. N. Güncan, Some operator inequalities associated with Kantorovich and Hölder-McCarthy inequalities and their applications. Turkish J. Math. 43 (2019), no. 1, 523–532.
- [3] F. A. Berezin, Covariant and contravariant symbols for operators. Math. USSR-Izv. 6 (1972), no. 5, 1117–1151.
- [4] M. T. Karaev, Berezin symbol and invertibility of operators on the functional Hilbert spaces. J. Funct. Anal. 238 (2006), no. 1, 181–192.
- [5] R. Tapdigoglu, New Berezin symbol inequalities for operators on the reproducing kernel Hilbert space. Oper. Matrices 15 (2021), no. 3, 1031–1043.



Non-Isotropic Approximation by Multidimensional Mellin-Type Singular Integral Operators: Asymptotic and Quantitative Results

Harun Karsli

Bolu Abant İzzet Baysal University, Department of Mathematics, Bolu, Türkiye
e-mail: karsli_h@ibu.edu.tr

Abstract

We introduce and investigate a family of multidimensional Mellin-type singular integral operators acting on the multiplicative group $\mathbb{R}_+^n = (0, \infty)^n$ endowed with a λ -non-isotropic metric structure. The proposed framework combines the multiplicative nature of Mellin analysis with a non-isotropic geometry that allows different scaling behaviors along different coordinate directions.

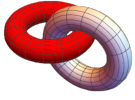
Under suitable normalization, localization, and logarithmic moment assumptions on the kernel family, we establish some convergence results of the operators and derive quantitative estimates in terms of the λ -non-isotropic distance. Furthermore, by means of a multidimensional Mellin–Taylor expansion, we derive an asymptotic expansion of the approximation process and prove a Voronovskaya-type theorem expressed through Mellin differential operators.

The obtained results provide a unified framework encompassing approximation, asymptotic behavior, and Voronovskaya-type results for multidimensional Mellin-type singular integral operators in non-isotropic settings.

Keywords: Mellin-type singular integral operators, non-isotropic approximation, asymptotic expansion, Voronovskaya theorem, quantitative approximation, multiplicative harmonic analysis.

References:

- [1] G. A. Anastassiou and A. Aral, Generalized Picard singular integrals. *Comput. Math. Appl.* 57 (2009), no. 5, 821–830.
- [2] L. Angeloni and G. Vinti, Variation and approximation in multidimensional setting for Mellin integral operators. In: *New Perspectives on Approximation and Sampling theory*, pp. 299–317. *Appl. Numer. Harmon. Anal.* Birkhäuser/Springer, Cham, 2014.
- [3] C. Bardaro, H. Karsli and G. Vinti, On pointwise convergence of linear integral operators with homogeneous kernels. *Integral Transforms Spec. Funct.* 19 (2008), no. 5-6, 429–439.
- [4] C. Bardaro, H. Karsli and G. Vinti, G. (2013). On pointwise convergence of Mellin type nonlinear m -singular integral operators. *Comm. Appl. Nonlinear Anal.* 20 (2013), no. 2, 25–39.
- [5] C. Bardaro and I. Mantellini, On Mellin convolution operators: a direct approach to the asymptotic formulae. *Integral Transforms Spec. Funct.* 25 (2014), no. 3, 182–195.
- [6] P. L. Butzer and S. Jansche, A direct approach to the Mellin transform. *J. Fourier Anal. Appl.* 3 (1997), no. 4, 325–376.
- [7] P. L. Butzer and S. Jansche, S. (1998). A self-contained approach to Mellin transform analysis for square integrable functions; applications. *Integral Transform. Spec. Funct.* 8 (1999), no. 3-4, 175–198.
- [8] R. Mamedov, *The Mellin Transform and Approximation Theory*. Elm, Baku, 1991.



A Dynamical Systems Approach to a Climate-Driven Aquatic Ecosystem with Fisheries Management

Hasen Mekki Öztürk⁽¹⁾ and Aytül Gökçe⁽²⁾

⁽¹⁾ *Ordu University, Department of Mathematics, Ordu, Türkiye*
e-mail: hasenozturk@gmail.com

⁽²⁾ *Ordu University, Department of Mathematics, Ordu, Türkiye*
e-mail: aytulgokce@odu.edu.tr

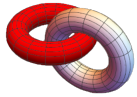
Abstract

The accumulation of greenhouse gases and subsequent environmental warming impose severe physiological stress on marine organisms and disrupt the structural balance of aquatic ecosystems, altering plankton and fish population dynamics [1–3]. To investigate these interacting stressors alongside human intervention, we formulate an autonomous ordinary differential equation model coupling greenhouse gas concentration, environmental temperature, and plankton and fish dynamics with an explicit fisheries management component. We analytically establish the well-posedness, positivity, and uniform boundedness of the system. We determine the algebraic conditions governing the local stability of the boundary equilibria, while evaluating the coexistence state numerically. Treating the atmospheric depletion rate of greenhouse gases, the management-targeted fish capacity, and the environmental cooling rate as primary bifurcation parameters, numerical continuation reveals that the stable coexistence equilibrium loses stability via supercritical Hopf bifurcations in all three cases. While variations in the depletion rate and management targets drive the system into sustained oscillatory regimes, a reduction in the environmental cooling rate forces the ecosystem through a period-doubling cascade. Positive maximal Lyapunov exponents and one-dimensional return maps are consistently indicative of a transition to chaotic dynamics driven by restricted thermal dissipation. Ultimately, these findings demonstrate that parameters influencing fish population dynamics play a critical role in ecosystem stability, highlighting the significance of properly calibrated fisheries management in maintaining ecological balance.

Keywords: Dynamical systems, stability analysis, numerical continuation, global warming, greenhouse gases.

References:

- [1] P. Kumar, V. Govindaraj, V. S. Erturk and M. S. Mohamed, Effects of greenhouse gases and hypoxia on the population of aquatic species: a fractional mathematical model. *Adv. Contin. Discrete Models* 2022, Paper No. 31, 19 pp.
- [2] S. Mandal, M. S. Islam, M. H. A. Biswas and S. Akter, A mathematical model applied to investigate the potential impact of global warming on marine ecosystems. *Appl. Math. Model.* 101 (2022), 19–37.
- [3] S. Sarker, A. K. Yadav, M. Akter, M. S. Hossain, S. R. Chowdhury, M. A. Kabir and S. M. Sharifuzaman, Rising temperature and marine plankton community dynamics: Is warming bad? *Ecological Complexity* 43 (2020), Paper No. 100857.



Delay-Induced Instability and Chaos in a Plankton-Fish Model Regulated by Toxins, Fear and Refuge

Hasen Mekki Öztürk

Ordu University, Department of Mathematics, Ordu, Türkiye
e-mail: hasenozturk@gmail.com

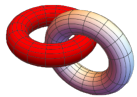
Abstract

Temporal lags in maturation are a recognised source of instability in aquatic food webs [3], yet their effect remains unexplored in a food chain already regulated by toxin-mediated mortality [1], predator fear [5] and a density-dependent refuge [2]. We extend a recent ordinary differential equation model of three species [4] comprising phytoplankton, zooplankton and fish by introducing a single discrete maturation delay into the fish recruitment term, representing the time between predation on zooplankton and its contribution to the adult fish stock. Using this delay as the primary bifurcation parameter, we analytically establish the well-posedness, positivity and uniform boundedness of solutions. The coexistence equilibrium, locally stable in the underlying ordinary differential equation system, loses stability through a supercritical Hopf bifurcation once the delay crosses a critical threshold. Numerical continuation beyond this threshold traces a cascade of period-doubling and fold bifurcations, and a positive maximal Lyapunov exponent together with a folded Poincaré section are consistent with a transition to chaotic dynamics at larger delays. These results illustrate how a biologically motivated maturation delay is sufficient to disrupt trophic stability, driving an otherwise well-regulated ecosystem from stable coexistence into dynamics consistent with chaos.

Keywords: Plankton-fish dynamics, stability analysis, maturation delay, bifurcation analysis, chaotic dynamics.

References:

- [1] J. Chattopadhyay, R. R. Sarkar, and S. Mandal, Toxin-producing plankton may act as a biological control for planktonic blooms-field study and mathematical modelling. *J. Theoret. Biol.* 215 (2002), no. 3, 333–344.
- [2] E. González-Olivares and R. Ramos-Jiliberto, Dynamic consequences of prey refuges in a simple model system: more prey, fewer predators and enhanced stability. *Ecological Modelling* 166 (2003), no. 1-2, 135–146.
- [3] Y. Kuang, *Delay Differential Equations with Applications in Population Dynamics*. Academic Press, Boston, 1993.
- [4] A. Mandal, N. Sk and S. Biswas, Nutrient enrichment and phytoplankton toxicity influence a diversity of complex dynamics in a fear-induced plankton-fish model. *J. Theoret. Biol.* 578 (2024), Paper No. 111698, 12 pp.
- [5] X. Wang, L. Zanette and X. Zou, Modelling the fear effect in predator-prey interactions. *J. Math. Biol.* 73 (2016), no. 5, 1179–1204.



On the Approximation of Borel Derivatives by Nonlinear Durrmeyer Type Operators

Hüseyin Erhan Altın⁽¹⁾ and Harun Karşlı⁽²⁾

⁽¹⁾ *Bolu Abant İzzet Baysal University, Department of Mathematics, Bolu, Türkiye*
e-mail: erhanaltin@ibu.edu.tr

⁽²⁾ *Bolu Abant İzzet Baysal University, Department of Mathematics, Bolu, Türkiye*
e-mail: karsli_h@ibu.edu.tr

Abstract

In the present study, we consider a sequence of nonlinear Durrmeyer type operators (ND_n) of the form

$$(ND_n)(f; x) = \int_0^1 K_n(x, t, f(t)) dt, \quad x \in [0, 1], \quad n \in \mathbb{N},$$

acting on Lebesgue measurable functions defined on $[0, 1]$ where $K_n(x, t, u)$ satisfy some suitable assumptions.

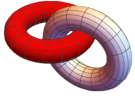
We give some approximation results with regard to the convergence of the operators ND_n to right, left, and symmetric Borel differentiable functions.

Our study extends some of the previous results about the linear integral operators and also aims to bring a new perspective to research in approximation with nonlinear operators by introducing a different concept of a derivative.

Keywords: Borel derivative, nonlinear operator, pointwise convergence.

References:

- [1] A. Sahai and G. Prasad, On the rate of convergence for modified Szász-Mirakyan operators on functions of bounded variation. *Publ. Inst. Math. (Beograd) (N.S.)* 53(67) (1993), 73–80.
- [2] C. Bardaro, J. Musielak and G. Vinti, *Nonlinear Integral Operators and Applications*. De Gruyter Ser. Nonlinear Anal. Appl., Vol. 9 Walter de Gruyter & Co., Berlin, 2003.
- [3] E. Ibikli, Approximation of Borel derivatives of functions by singular integrals. *Trans. Acad. Sci. Azerb. Ser. Phys.-Tech. Math. Sci., Math. Mech.*, 21 (2001), no. 4, 72–76.
- [4] H. Karşlı and I. U. Tiriyaki, Approximation of Borel derivatives of functions via non-linear singular integral operators. *Palest. J. Math.* 10 (2021), no. 2, 673–684.
- [5] J. Musielak, On some approximation problems in modular spaces. In: *Constructive function theory '81* (Varna, 1981), pp. 455–461. Publishing House of the Bulgarian Academy of Sciences, Sofia, 1983.
- [6] H. Karşlı, On convergence of a sequence of nonlinear Durrmeyer operators. *PanAmer. Math. J.* 24 (2014), no. 4, 1–13.



Arithmetic and Measure Theory

Ileana Bucur

Civil Engineering Faculty Bucharest, București, Romania

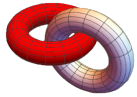
Abstract

In this paper we introduce so called “standard sequences” of natural numbers i.e. the sequences $p = (p_n)_n$ of increasing numbers such that for any $k \in \mathbb{N}^*$ we have:

$$\sum_{i>k} \frac{1}{p_i} \leq \frac{1}{p_k(p_k - 1)}, \quad p_1 \geq 2.$$

The set S of all such kind of sequences endowed with anti-lexicographic order becomes isomorphic with the unit interval $(0, 1]$.

Then we consider a special order interval in S and we use them in proving the analyticity with respect to a given family of subsets of an arbitrary non-empty set.



Hardy-Rogers Type Fixed Point Results for Quantale-Valued Quasi-Metric Spaces

Irem Eroğlu

Ordu University, Department of Mathematics, Ordu, Türkiye
e-mail: iremeroglu@odu.edu.tr

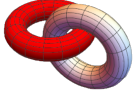
Abstract

Metric spaces have been generalized by modifying the classical metric axioms. For example, by removing the symmetry condition, quasi-metric spaces were described in [3]. Another important generalization of metric spaces is partial metrics and these structures were introduced by Matthews [6]. Partial metric spaces play an important role in computer science. In 1977, Flagg [2] defined the quantale-valued quasi metric spaces by changing the set $[0, \infty]$ with a value-quantale. In [1], the notion of an action was defined and Banach contraction principle was extended to the setting of quantale-valued quasi-metric spaces. These structures are generalization of both metric spaces and probabilistic metric spaces [8], and they are important tools in quantitative domain theory and denotational semantics. In [4], Hardy and Rogers obtained an important generalization of Reich's fixed point result given in [7]. In this work, we first present some important topological properties of quantale-valued quasi metric spaces. Then, we extend the Hardy-Rogers type fixed point results to the setting of quantale-valued quasi-metric spaces and we derive some consequences in both probabilistic and partial quantale-valued metric spaces [5]. Our results generalize the results given in the literature.

Keywords: Value-quantale, fixed point, complete lattice, action.

References:

- [1] B. Flagg, R. Kopperman, The asymmetric topology of computer science in: S. Brookes, et al., (Eds.), *Mathematical Foundations of Programming Language Semantics*, Lect. Notes Comput. Sci. 802 (1993), 544–553.
- [2] R. C. Flagg, Quantales and continuity spaces. *Algebra Universalis* 37 (1997), no. 3, 257–276.
- [3] P. Fletcher and W. F. Lindgren, *Quasi-Uniform Spaces*. Marcel Dekker, New York, 1982.
- [4] G. E. Hardy and T. D. Rogers, A generalization of a fixed point theorem of Reich. *Canad. Math. Bull.* 16 (1973), 201–206.
- [5] R. D. Kopperman, S. Matthews and H. Pajoohes, Partial metrizability in value quantales. *Appl. Gen. Topol.* 5 (2004), no. 1, 115–127.
- [6] S. G. Matthews, Partial metric topology. *Proc. 8th summer conference on topology and its applications*, ed S. Andima et al., *Annals of the New York Academy of Sciences*, New York 728 (1994), 183–197.
- [7] S. Reich, Kannan's fixed point theorem. *Boll. Un. Mat. Ital.* (4) 4 (1971), 1–11.
- [8] B. Schweizer and A. Sklar, *Probabilistic Metric Spaces*, North-Holland Series in Probability and Applied Mathematics, Elsevier Science Publishing Co. Inc., 1983.



Multidimensional Max-Min Sampling Operators with Applications

İsmail Aslan⁽¹⁾ and Wolfgang Erb⁽²⁾

⁽¹⁾ *Hacettepe University, Department of Mathematics, Ankara, Türkiye*
e-mail: ismail-aslan@hacettepe.edu.tr

⁽²⁾ *University of Padua, Department of Mathematics “Tullio Levi-Civita”, Padua, Italy*
e-mail: wolfgang.erb@unipd.it

Abstract

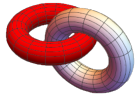
In this study, we investigate multidimensional max-min type sampling operators, including neural network operators [1]. We establish both pointwise and uniform convergence for these operators and analyze their rate of approximation. Finally, we demonstrate that these operators can function as morphological operations, such as dilation and erosion, which serve as the fundamental building blocks of morphological image processing.

Acknowledgement This research is supported by the Scientific and Technological Research Council of Türkiye (TÜBİTAK) under grant number 124F356.

Keywords: Sampling operators, neural network operators, image processing.

References:

- [1] İ. Aslan, Approximation by max-min neural network operators. *Numer. Funct. Anal. Optim.* 46 (2025), no. 4-5, 374–393.



A Generalization of Banach's Closed Range Theorem

Krzysztof Zakrzewski

Warsaw School of Economics, Collegium of Economic Analysis, Warsaw, Poland
e-mail: kzakrze@sgh.waw.pl

Abstract

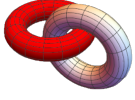
For a real topological vector space X , let X^* denote its dual space, that is the space of linear continuous functionals endowed with the *weak** topology. For a continuous linear operator $T : X \rightarrow Y$ between two topological vector spaces, by $T^* : Y^* \rightarrow X^*$ we denote the dual operator of T .

A classical theorem of Banach states that a continuous linear operator $T : X \rightarrow Y$ between two Banach spaces has closed range if and only if its dual operator $T^* : Y^* \rightarrow X^*$ has closed range. A generalization of this theorem to the setup of general locally convex topological vector spaces can be found in [1] [Theorem 7.3/Chapter IV]. This generalization states that a continuous linear operator $T : X \rightarrow Y$ between two locally convex topological vector spaces is *weak – weak* open onto its image if and only if T^* has closed range. The main theorem proved during this talk is a symmetric generalization of Banach's closed range theorem: a continuous linear operator $T : X \rightarrow Y$ between two locally convex topological vector spaces has closed range if and only if T^* is *weak* – weak** open onto its image.

Keywords: Dual operator, closed range theorem, topological vector spaces.

References:

- [1] H. H. Schaefer, Topological Vector Spaces. Springer Verlag, New York, 1971.



ℓ_0 -Regularized Random Graphical Modeling for Sparse Microbiome Network Inference on AGP and HMP Data

Mahdieh Farzin Asanjan⁽¹⁾, Vilda Purutcuoğlu⁽²⁾ and Ernst Wit⁽³⁾

⁽¹⁾ *Department of Biomedical Engineering, Middle East Technical University, Ankara, Türkiye*
e-mail: mah.farzin2010@gmail.com

⁽²⁾ *Department of Statistics, Middle East Technical University, Ankara, Türkiye*
e-mail: vpurutcu@metu.edu.tr

⁽³⁾ *Institute of Computational Science, Università della Svizzera Italiana, Lugano, Switzerland*
e-mail: e.c.wit@rug.nl

Abstract

Microbiome interaction network inference is a challenging problem due to high dimensionality, compositional constraints, and sparsity observed in microbial abundance data. In graphical modeling frameworks, microbial interactions are represented through conditional dependency structures encoded by the precision matrix $\Theta = \Sigma^{-1}$, where zero off-diagonal elements indicate conditional independence relationships between microbial taxa.

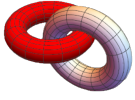
In this study, we investigate sparse precision matrix estimation within an ℓ_0 -regularized Random Graphical Modeling (RGM) framework for microbiome network inference. The microbial interaction network is represented as a random graph $G = (V, E)$, where V denotes microbial taxa and E represents microbial interactions. Unlike conventional Gaussian graphical models based on ℓ_1 -penalized graphical lasso approaches, the proposed framework promotes exact sparsity by directly penalizing the number of nonzero elements in the precision matrix. The estimation procedure is formulated through the penalized likelihood optimization problem

$$\log |\Theta| - \text{tr}(S\Theta) - \lambda \|\Theta\|_0,$$

where S denotes the sample covariance matrix, $\text{tr}(S\Theta)$ measures data fidelity, and the ℓ_0 penalty explicitly controls graph sparsity. The sparsity structure is modeled probabilistically through an edge inclusion parameter and estimated under sparse graphical assumptions. Since ℓ_0 -regularized optimization leads to a non-convex optimization problem, sparse-step approximation techniques are employed to obtain stable sparse solutions. The proposed methodology is compared against SPIEC-EASI and conventional ℓ_1 -based graphical modeling approaches for sparse microbial network inference. Previous studies demonstrated promising performance of the proposed ℓ_0 -based framework on simulated toy datasets under different sparsity scenarios. In this work, the method is applied for the first time to large-scale real microbiome datasets, including stool samples from the American Gut Project and multiple body-site samples from the Human Microbiome Project. A unified preprocessing pipeline involving filtering, centered log-ratio transformation, and variance-based feature selection is employed to ensure consistent comparison across methods.

Experimental results indicate that the proposed framework captures sparse topological structures more effectively than conventional ℓ_1 -based graphical models, producing more interpretable and biologically plausible microbial interaction networks while preserving core conditional dependency relationships. The findings suggest that sparsity-aware random graphical modeling provides a flexible mathematical framework for high-dimensional microbiome network inference.

Keywords: Microbiome networks, random graphical models, graphical lasso, SPIEC-EASI.



Consistent Parameter Estimation for Stochastic Differential Equations via Theta-Milstein GMM: Theory, Stability Thresholds, and Empirical Validation

Majda El Allali⁽¹⁾, Toufik Chaayra⁽¹⁾, Mohammed Khellouf⁽¹⁾ and Zakaria El Allali⁽¹⁾

⁽¹⁾ Team of MSC, Department of Mathematics, FPN, Mohammed First University in Oujda, Morocco

e-mail: majda.elallali@ump.ac.ma

e-mail: t.chaayra@ump.ac.ma

e-mail: mohammed.khellouf@ump.ac.ma

e-mail: z.elallali@ump.ac.ma

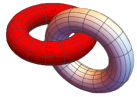
Abstract

We establish the L^2 -consistency of generalized method of moments (GMM) parameter estimators for stochastic differential equations discretized via θ -Milstein schemes. Our key theoretical contribution identifies the unconditional mean-square A -stability threshold: $\theta \geq 3/2$, strictly above the classical $\theta \geq 1/2$ for Euler-Maruyama methods. This elevation stems from variance amplification by the second-order Itô correction. We derive explicit GMM estimators exploiting exact conditional moments and prove L^2 -consistency via moment decomposition. Geometric Brownian motion illustrates lower-order scheme failures: under high volatility, Euler-based maximum likelihood estimation (Euler-MLE) generates negative trajectories in 70.7% of paths at coarse steps. Monte Carlo experiments across three volatility regimes confirm convergence rates and demonstrate substantially lower root-mean-square error (RMSE) than Euler-MLE and standard Milstein schemes. Empirical application to 495 daily S&P 500 prices yields 53% mean-squared error (MSE) reduction versus Euler-MLE, with coefficients of determination exceeding 98.9%.

Keywords: L^2 -Consistency, θ -Milstein scheme, A -stability, generalized method of moments, parameter estimation, stochastic differential equations, S&P 500 index

References:

- [1] Y. Mahraz, T. Chaayra, H. El Ouardani and Z. El Allali, On consistency Gaussian estimators for discretely observed G-Brownian motion stochastic process with application to real long-term rate dynamics. *J. Indian. Soc. Probab. Stat.* 26 (2025), 763–786.
- [2] M. Bayram, G. Orucova Buyukoz and T. Partal, Parameter estimation in a Black-Scholes model. *Thermal Science* 22 (2018), no. 1, 117–122.
- [3] E. Buckwar and T. Sickenberger, A comparative linear mean-square stability analysis of Maruyama- and Milstein-type methods, *Math. Comput. Simulation* 81 (2011), no. 6, 1110–1127.



Existence Results for a Neumann Problem with Hardy-Littlewood-Sobolev Critical Exponent and Hardy Potential

Manal Seddar⁽¹⁾ and Ali Rimouche⁽²⁾

⁽¹⁾ *Laboratoire Systèmes Dynamiques et Applications (SDA), University of Tlemcen, Department of Mathematics, Tlemcen, Algeria*
e-mail: manel.seddar@univ-tlemcen.dz

⁽²⁾ *Laboratoire Systèmes Dynamiques et Applications (SDA), University of Tlemcen, Department of Mathematics, Tlemcen, Algeria*
e-mail: ali.rimouche@univ-tlemcen.dz

Abstract

We study a class of nonlinear elliptic equations involving a Hardy potential and a critical Choquard-type nonlocal term. The presence of both the inverse-square potential and the critical nonlocal nonlinearity introduces significant analytical difficulties.

Using variational methods, we prove the existence of a nontrivial weak solution by applying the Mountain Pass Theorem. A key step in the analysis is the verification of the Palais–Smale condition below a suitable critical level.

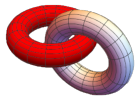
To establish the subcritical nature of the energy level, we construct appropriate test functions and perform asymptotic expansions near the boundary. This allows us to show that the Mountain Pass level lies strictly below the critical threshold.

As a consequence, we obtain the existence of a nontrivial solution to the problem.

Keywords: Elliptic problems, Choquard equations, critical Hardy–Littlewood-Sobolev exponent, subcritical perturbation.

References:

- [1] H. Brézis and L. Nirenberg, Positive solutions of nonlinear elliptic equations involving critical Sobolev exponents. *Comm. Pure Appl. Math.* 36 (1983), no. 4, 437–477.
- [2] E. H. Lieb, Sharp constants in the Hardy-Littlewood-Sobolev and related inequalities. *Ann. of Math.* (2) 118 (1983), no. 2, 349–374.
- [3] V. Moroz and J. Van Schaftingen, A guide to the Choquard equation. *J. Fixed Point Theory Appl.* 19 (2017), no. 1, 773–813.
- [4] J. Giacomoni, S. Rawat and K. Sreenadh, Critical exponent Neumann problem with Hardy-Littlewood-Sobolev nonlinearity. *arXiv:2304.05447* (2023).
- [5] G. Gu and A. Jevnikar, Nonlocal problems with Hardy-Littlewood-Sobolev critical exponent and Hardy potential. *arXiv:2509.15697* (2025).



Jakimovski-Leviatan Type Operators Including Bessel's K Function in the Kernel

Mehmet Ali Özarslan

*Eastern Mediterranean University, Faculty of Arts and Sciences, Department of Mathematics,
Gazimagusa, TRNC, Mersin 10, Türkiye
e-mail: mehmetali.ozarslan@emu.edu.tr*

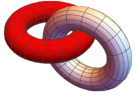
Abstract

In this paper we introduce Jakimovski-Leviatan type operators including Bessel's K function in the kernel. These operators are the composition of the exponential type operators connected with x^3 and the Jakimovski-Leviatan operators. We investigate convergence properties of these operators using Korovkin type approximation theorems. We also obtain quantitative type result by introducing appropriate Lipschitz type functionals including cubic polynomial weight. We further obtain the rate of convergence of the operators by means of modulus of continuity.

Keywords: Jakimovski-Leviatan operators, exponential type operators, Lipschitz type functional.

References:

- [1] U. Abel, Complete asymptotic expansion for an exponential-type operator related to $p(x) = x^3$. Anal. Math. 48 (2022), no. 4, 913–924.
- [2] T. Acar, A. Aral, D. Cárdenas-Morales and P. Garrancho, Szász-Mirakyan type operators which fix exponentials. Results Math. 72 (2017), no. 3, 1393–1404.
- [3] V. Gupta, Approximation with certain exponential operators. Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. RACSAM 114 (2020), no. 2, Paper No. 51, 15 pp.
- [4] V. Gupta, A. Aral and C. V. Muraru, Modification of exponential type operators preserving exponential functions connected with x^3 . Mediterr. J. Math. 18 (2021), no. 5, Paper No. 222, 14 pp.
- [5] A. Holhoş, Exponential-type operators generated by power functions. Bull. Iranian Math. Soc. 51 (2025), no. 5, Paper No. 64, 13 pp.
- [6] M. E. H. Ismail and C. P. May, On a family of approximation operators. J. Math. Anal. Appl. 63 (1978), no. 2, 446–462.
- [7] A. Jakimovski and D. Leviatan, Generalized Szász operators for the approximation in the infinite interval. Mathematica (Cluj) 11(34) (1969), 97–103.
- [8] M. A. Özarslan, Approximation properties of Jain-Appell operators. Appl. Anal. Discrete Math. 14 (2020), no. 3, 654–669.



On the Space of Sequential Modulus Approximately Continuous Function

H. Sabor Behmanush and Mehmet Küçükaslan

Mersin University, Department of Mathematics, Mersin, Türkiye
e-mail: h.s.behmanush1989@gmail.com
e-mail: mkkaslan@gmail.com; mkucukaslan@mersin.edu.tr

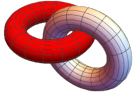
Abstract

In this paper, we study the space of sequentially γ -modulus approximately continuous functions with respect to a fixed sequence $\bar{t} = \{t_n\} \in \mathbb{S}$. We first show that the class $\mathcal{SA}_{\gamma, \bar{t}}$ of all sequentially γ -modulus approximately continuous real-valued functions forms a vector space under pointwise addition and scalar multiplication. Then, we consider the bounded subspace $\mathcal{SA}_{\gamma, \bar{t}}^\infty$ and prove that it becomes a normed linear space under the supremum norm: $\|f\|_\infty = \sup_{x \in \mathbb{R}} |f(x)|$. Moreover, we show that this space is complete, and hence it is a Banach space. We also compare this space with the classical spaces of continuous functions and bounded continuous functions. In particular, we prove that: $C(\mathbb{R}) \subset \mathcal{SA}_{\gamma, \bar{t}}$ and $C_b(\mathbb{R}) \subset \mathcal{SA}_{\gamma, \bar{t}}^\infty$ but show that these inclusions are proper. This demonstrates that sequential γ -modulus approximate continuity provides a broader framework than classical continuity, allowing functions to exhibit sparse exceptional oscillations while preserving a rigorous density-type limiting behavior.

Keywords: Lebesgue measure, density point, Lebesgue density theorem, modulus density point.

References:

- [1] C. Goffman, C. J. Neugebauer and T. Nishiura, Density topology and approximate continuity. *Duke Math. J.* 28 (1961), 497–505.
- [2] C. Goffman and D. Waterman, Approximately continuous transformations. *Proc. Amer. Math. Soc.* 12 (1961), 116–121.
- [3] O. Haupt and C. Pauc, La topologie de Denjoy approximative envisagée comme vraie topologie. *C. R. Acad. Sci. Paris* 234 (1952), 390–392.
- [4] M. Terepeta, A new approach to ψ -continuity. *Tatra Mt. Math. Publ.* 42 (2009), 107–117



Approximate Formula for Cramér Coefficient in a Non-Linear Insurance Model Under Gamma-Distributed Claims

Melike Berna Ozkan⁽¹⁾, Basak Geve Ekinci⁽²⁾, Zulfiye Hanalioglu⁽³⁾ and Tahir Khaniyev⁽⁴⁾

⁽¹⁾ *TOBB University of Economics and Technology, Department of Industrial Engineering, Ankara, Türkiye*
e-mail: melikeberna.ozkan@etu.edu.tr

⁽²⁾ *Turkish Aeronautical Association University, Department of Industrial Engineering, Ankara, Türkiye*
e-mail: bgever@thk.edu.tr

⁽³⁾ *Karabuk University, Department of Actuarial Sciences, Karabuk, Türkiye*
e-mail: zulfiyyamammadova@karabuk.edu.tr

⁽⁴⁾ *TOBB University of Economics and Technology, Department of Industrial Engineering, Ankara, Türkiye*
e-mail: tahirkhaniyev@etu.edu.tr

Abstract

This paper provides an in-depth investigation of non-linear Cramér–Lundberg type insurance models, which occupy a critical position in risk theory and actuarial modeling, and presents Cramér-type approximate results for the ruin probability of the system. Although the Cramér coefficient (C) and the adjustment coefficient (r) used in insurance models are well known in the literature [1,2], their existing analytical structures are not directly suitable for computation under general distributions such as the Gamma distribution due to non-linear effects, complex integral transformations, and distributional difficulties.

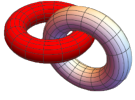
Within the scope of this study, the ruin probability of the non-linear model has been analyzed under the assumption that claim sizes follow a Gamma distribution, and a practical approximate formula suitable for the direct computation of the Cramér coefficient has been developed. While resolving the stochastic structure of the process, renewal theory, approximation approaches, and risk process techniques were utilized simultaneously.

In the final part of the paper, several special cases are examined, and the numerical performance of the theoretical results is demonstrated through applied examples. The proposed method is expected to provide an innovative contribution to the literature in terms of risk analysis and actuarial decision-making for insurance systems involving non-linear premium structures.

Keywords: Non-linear insurance model, Approximate formula for the Cramér coefficient, Gamma-distributed claims.

References:

- [1] T. Mikosch, Non-Life Insurance Mathematics: An Introduction with the Poisson Process. Springer-Verlag, Berlin-Heidelberg, Germany, 2004.
- [2] B. Geve, Z. Hanalioglu and T. Khaniyev T. (2025). An approximate formula for computation of adjustment coefficient of a non-linear Cramér–Lundberg risk model with gamma claims. Methodol. Comput. Appl. Probab. 27 (2025), Article No. 64, 19 pp.



On the Finite Element Approximation in the L^∞ - Norm of a Class of Semi-Linear Elliptic Systems of QVIs

Messaoud Boulbrachene

Sultan Qaboos University, Muscat, Oman
e-mail: boulbrac@squ.edu.om

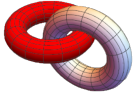
Abstract

In this work, we are concerned with the standard finite element approximation of a class of systems of elliptic quasi-variational inequalities (QVIs) with nonlinear source terms arising in stochastic control theory (see [1, 2]). The presence of nonlinear source terms further increases the mathematical complexity of the problem and makes the analysis of numerical approximations in the L^∞ -norm more challenging. By means of the concept of subsolutions, we establish the optimal convergence order of the approximation without requiring the assumption made in ([3], (2.6), p. 431), thereby encompassing a broader class of systems.

Keywords: Semilinear system, quasi-variational inequalities, subsolutions, finite element, L^∞ -error estimate.

References:

- [1] P.-L. Lions and J.-L. Menaldi, Optimal control of stochastic integrals and Hamilton-Jacobi-Bellman equations (part I). SIAM J. Control Optim. 20 (1982), no. 1, 58–81.
- [2] L. C. Evans, Classical solutions of the Hamilton-Jacobi-Bellman equation for uniformly elliptic operators. Trans. Amer. Math. Soc. 275 (1983), no. 1, 245–255.
- [3] M. Boulbrachene, L^∞ -error estimate for a class of semilinear systems of quasi-variational inequalities, J. Nonlinear Var. Anal. 5 (2021), no. 3, 429–440.



Spline-Based Laplace-Stieltjes Approximation for Saturated Generalized-Gamma Aggregates

Mohammad Oumoucha⁽¹⁾, Faissal El Bouanani⁽¹⁾, Zakaria El Allali⁽²⁾ and Moncef Zarrouk⁽¹⁾

⁽¹⁾ *ENSIAS, Mohammed V University in Rabat, Morocco*
e-mail: mohammad.oumoucha@um5.ac.ma
e-mail: f.elbouanani@um5s.net.ma
e-mail: moncef_zarrouk@um5.ac.ma

⁽²⁾ *Team of MSC, Department of Mathematics, FPN, Mohammed First University in Oujda, Morocco*
e-mail: z.elallali@ump.ac.ma

Abstract

Generalized-Gamma (GG) distributions provide a flexible framework for modeling fading, reliability, and aggregate uncertainty in engineering systems. In many practical settings, however, the effective contribution of each component is subject to saturation or clipping, leading to aggregate variables of the form.

$$S_N = \sum_{i=1}^N a_i \min(G_i, T_i),$$

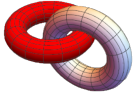
where G_i follows a GG distribution, $a_i > 0$ is a deterministic weight, and T_i is a saturation threshold. Unlike ordinary sums of GG variables, each saturated component has a mixed distribution, with a continuous part on $[0, T_i)$ and a point mass at T_i , which makes the distributional analysis of SN nontrivial.

This work proposes a cubic-spline-assisted Laplace–Stieltjes transform approximation for the distribution of such saturated aggregates. The point mass induced by saturation is treated exactly in the transform domain, while cubic splines approximate the continuous GG density over the unsaturated region. This leads to finite polynomial-exponential integral expressions for the component transforms. By independence, the transform of S_N is obtained as a product of the weighted component transforms, enabling efficient evaluation of the probability density function and cumulative distribution function, and outage-type performance metrics. The proposed approach is applied to outage analysis of clipped diversity receivers over GG fading channels. Numerical comparisons with Monte Carlo simulations show close agreement across representative fading and saturation regimes, confirming that the spline-assisted Laplace–Stieltjes approximation provides a stable and accurate tool for performance analysis of saturated stochastic systems.

Keywords: Generalized-Gamma distribution, cubic spline approximation, Laplace–Stieltjes transform, saturated random variables, outage probability, wireless fading.

References:

- [1] E. W. Stacy, A generalization of the gamma distribution. *Ann. Math. Statist.* 33 (1962), 1187–1192.
- [2] T. Chaayra, H. Ben-Azza and F. El Bouanani, New accurate approximation for the sum of generalized gamma distributions and its applications. *Appl. Math. Inf. Sci.* 14 (2020), no. 6, 1121–1136.
- [3] E.-H. El Baqqaly, F. El Bouanani and Z. El Allali, Cubic spline-based approximation for the sum of i.i.d. lognormal distributions. In *Proc. 5th International Conference on Advanced Communication Technologies and Networking (CommNet)*, Marrakech, Morocco, 2022, pp. 1–7.



$\mathcal{L}^2$ -Consistent and Asymptotically Normal Quasi-Maximum Likelihood Estimators for Milstein-Discretized Scalar Diffusion Process

Mohammed Khellouf⁽¹⁾, Toufik Chaayra⁽¹⁾, Majda El Allali⁽¹⁾ and Zakaria El Allali⁽¹⁾

⁽¹⁾Team of MSC, Department of Mathematics, FPN, Mohammed First University in Oujda, Morocco

e-mail: mohammed.khellouf@ump.ac.ma

e-mail: t.chaayra@ump.ac.ma

e-mail: majda.elallali@ump.ac.ma

e-mail: z.elallali@ump.ac.ma

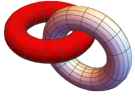
Abstract

Although the Milstein scheme achieves strong-order-1 pathwise accuracy for stochastic differential equations, the $\mathcal{L}^2$ -consistency and asymptotic distribution of parameter estimators derived from it have not been rigorously established. This paper closes that gap by developing a quasi-maximum likelihood estimation (QMLE) framework for Milstein-discretized scalar diffusion, using geometric Brownian motion as a benchmark. Drift and volatility estimators are constructed via martingale estimating functions on conditional moments of the Milstein scheme, avoiding evaluation of the exact transition density. $\mathcal{L}^2$ -consistency is proved under in-fill asymptotics using Gaussian moment identities and the Marcinkiewicz-Zygmund inequality; asymptotic normality is established with closed-form asymptotic variances, and joint Wald confidence regions are justified. Monte Carlo simulations show that Milstein-QMLE achieves lower absolute error than Euler-Maruyama-based MLE (Euler-MLE) on coarse grids; under high volatility, Euler-MLE fails positivity preservation in 70.7% of paths versus 0.0% for Milstein-QMLE. An empirical S&P 500 application yields a 27% reduction in out-of-sample mean-squared error.

Keywords: $\mathcal{L}^2$ -Consistency, asymptotic normality, Milstein scheme, quasi-maximum likelihood, parameter estimation, empirical Analysis.

References:

- [1] Y. El Ansari, T. Chaayra, F. El Bouanani, L. Omari and M. Amrouch, Strong consistency estimators of the Brennan-Schwartz diffusion process based on martingales approach. Stat 11 (2022), Paper No. e499, 20 pp.
- [2] A. K. Sinha (2021). The reliability of geometric Brownian motion forecasts of S&P500 index values. J. Forecast. 40 (2021), no. 8, 1444–1462.
- [3] Y. Mahraz, T. Chaayra, H. El Ouardani and Z. El Allali, On consistency Gaussian estimators for discretely observed G-Brownian motion stochastic process with application to real long-term rate dynamics. J. Indian Soc. Probab. Stat. 26 (2025), 763–786.



Approximation Laws for Optimal Stopping Under Generalized-Gamma Uncertainty

Moncef Zarrouk⁽¹⁾, Mohamed Oumoucha⁽¹⁾, Faissal El Bouanani⁽¹⁾, and Zakaria El Allali⁽²⁾

⁽¹⁾ *ENSIAS, Mohammed V University in Rabat, Morocco*
e-mails: moncef.zarrouk@um5.ac.ma
e-mails: mohammad.oumoucha@um5.ac.ma
e-mails: f.elbouanani@um5s.net.ma

⁽²⁾ *The Multidisciplinary Faculty of Nador, Mohamed First University, Oujda, Morocco*
e-mail: z.elallali@ump.ac.ma

Abstract

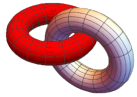
This work develops finite approximation laws for a class of optimal-stopping problems under generalized-gamma uncertainty. A finite sequence of candidates is observed sequentially, with each candidate associated with a random performance variable governed by a generalized-gamma distribution. The objective is to determine an optimal stopping rule for accepting a candidate while controlling observation costs and terminal failure risk. By exploiting the flexibility of the generalized-gamma model, we derive tractable expressions for the stopping probability, failure probability, expected stopping time, and expected observation cost. The resulting formulas provide a compact analytical description of the tradeoff between distributional shape, decision quality, and sampling effort. Numerical illustrations validate the proposed approximation laws and show how the generalized-gamma parameters affect stopping behavior.

Keywords: Stochastic optimization, optimal stopping, generalized-gamma distribution, uncertainty modeling, reliability analysis, approximation laws.

Acknowledgement: This publication is based upon work from COST Action Randomised Optimisation Algorithms Research Network (ROAR-NET), CA22137, supported by COST (European Cooperation in Science and Technology).

References:

- [1] E. W. Stacy, A generalization of the gamma distribution. *Ann. Math. Statist.* 33 (1962), 1187–1192.
- [2] G. Peskir and A. Shiryaev, *Optimal Stopping and Free-Boundary Problems*. Birkhäuser, Basel, 2006.
- [3] M. Zarrouk, F. El Bouanani and Z. El Allali, Optimal stopping-based online node selection for multi-parameter federated learning in IoT networks under multiuser interference. *IEEE Trans. Machine Learning Commun. Networking* 4 (2026), 391–408.



Operator-Theoretic Extensions of Hermite-Hadamard Type Inequalities

Muhammad Muddassar

*University of Engineering and Technology Taxila,
Department of Mathematical Sciences, Taxila 47050, Pakistan
e-mail: muhammad.muddassar@uettaxila.edu.pk*

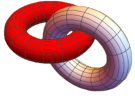
Abstract

In this article, we introduce a unified operator-theoretic framework for Hermite-Hadamard type inequalities by employing pseudo-integral operators and associated differential operators. The proposed approach extends several known results involving classical, h -, (α, m) -, and $(h - (\alpha, m))$ -convex functions. New bounds are established for functions whose generalized derivatives satisfy operator convexity conditions. Applications to fractional-type means and numerical integration formulas are also discussed, demonstrating that the obtained results recover and improve a variety of existing inequalities as particular cases.

Keywords: Hermite-Hadamard inequality, pseudo-integral operator, pseudo-differential operator, generalized convexity, operator inequalities, nonlocal analysis.

References:

- [1] N. Azzouz, B. Benaissa, H. Budak and I. Demir, Hermite-Hadamard-Mercer type inequalities for fractional integrals: a study with h -convexity and ψ -Hilfer operators. Bound Value Probl. 2025, Paper No. 14, 23 pp.
- [2] N. Azzouz and B. Benaissa, Exploring Hermite-Hadamard-type inequalities via ψ -conformable fractional integral operators. J. Ineq. Math. Anal. 1 (2025), no. 1, 15–27.
- [3] S. Ozcan and I. İşcan, Some new Hermite-Hadamard type inequalities for s -convex functions and their applications. J Inequal Appl 2019, Paper No. 201, 11 pp.
- [4] N. Dunford and J. T. Schwartz, Linear Operators (Part-I: General Theory), Interscience Publishers, Inc., New York, 1957.



An Accurate Method for Solving Fractional Integro-Differential Equations

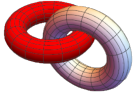
Muhammed I. Syam

United Arab Emirates University, Department of Mathematical Sciences, Al Ain, UAE
e-mail: m.syam@uaeu.ac.ae

Abstract

This talk presents analytical and numerical techniques for solving a class of fractional integro-differential equations. A modified operational matrix method is then developed to approximate the solution by constructing operational matrices for the fractional differential and integral operators and transforming the original problem into a system of algebraic equations. The proposed approach eliminates the need for projection procedures while providing high computational efficiency and accuracy. Several numerical examples are included to validate the performance of the method. The numerical solutions exhibit excellent agreement with the corresponding exact solutions, and the approximate fractional solutions converge to the classical integer-order solutions as the fractional order approaches one. The accuracy of the proposed algorithm is assessed using absolute and L_2 -norm errors, demonstrating errors of very small magnitude. In addition, the influence of the fractional-order parameter on the solution behavior is investigated. Finally, the Laplace transform is employed to derive exact solutions for several classes of homogeneous fractional integro-differential equations. The analytical results further confirm that the fractional solutions continuously approach the corresponding classical solutions as the fractional order tends to one, thereby validating both the theoretical analysis and the proposed numerical method.

Keywords: Fractional integro-differential equations, operational matrix method, Laplace transform.



A Characterization of Distributional Convergence with Respect to Power Series Methods

Mustafa Gülfirat⁽¹⁾ and Mehmet Ünver⁽²⁾

⁽¹⁾ *Ankara University, Department of Mathematics, Ankara, Türkiye*
e-mail: mgulfirat@ankara.edu.tr

⁽²⁾ *Ankara University, Department of Mathematics, Ankara, Türkiye*
e-mail: munver@ankara.edu.tr

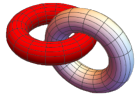
Abstract

In this talk, we introduce a notion of P -distributional convergence. Using the known characterization of P -strong convergence in terms of P -statistical convergence and P -uniform integrability, we obtain an equivalent characterization involving P -distributional convergence. Consequently, we establish the equivalence between P -strong convergence, P -statistical convergence together with P -uniform integrability, and P -distributional convergence together with P -uniform integrability. For the corresponding characterization in the topological setting, we refer the interested reader to Theorem 4 in [3].

Keywords: P -statistical convergence, P -strong convergence, P -distributional convergence.

References:

- [1] M. K. Khan and C. Orhan, Characterizations of strong and statistical convergences. *Publ. Math. Debrecen* 76 (2010), no. 1-2, 77–88.
- [2] M. Ünver and C. Orhan, Statistical convergence with respect to power series methods and applications to approximation theory. *Numer. Funct. Anal. Optim.* 40 (2019), no. 5, 535–547.
- [3] M. Ünver and N. Şahin Bayram, On statistical convergence with respect to power series methods. *Positivity* 26 (2022), no. 3, Paper No. 55, 13 pp.



Szász-Durrmeyer Type Degenerate Hermite Operators

Mustafa Kara and Mehmet Ali Özarslan

*Eastern Mediterranean University, Faculty of Arts and Sciences, Department of Mathematics,
Gazimagusa, TRNC, Mersin 10, Türkiye
e-mail: mustafa.kara@emu.edu.tr
e-mail: mehmetali.ozarslan@emu.edu.tr*

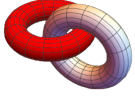
Abstract

In this paper, we introduce a new class of Szász-Durrmeyer type positive linear operators constructed by combining the parametric structure of degenerate Hermite polynomials with integral kernels of Gamma type, providing a flexible generalization of several classical approximation processes. We first derive explicit formulas for the moments and central moments of the operators, which play a fundamental role in the analysis of their approximation behavior. By using Korovkin-type theorems on unbounded intervals, the uniform convergence of the operators to the target function is established in suitable weighted spaces. Furthermore, quantitative estimates for the rate of convergence are obtained by means of the classical modulus of continuity, second-order modulus of smoothness, Peetre's K -functional, and Ditzian-Totik type tools. Voronovskaya-type asymptotic results are also investigated. In addition, graphical simulations and numerical examples are presented to illustrate the convergence behavior of the operators for different parameter values. The results demonstrate that the introduced family provides an effective and natural extension of classical Szász-type operators within the framework of degenerate Hermite polynomials.

Keywords: Degenerate Hermite polynomials, Szász-Durrmeyer type operators, rate of convergence, weighted approximation.

References:

- [1] G. Krech, A note on some positive linear operators associated with the Hermite polynomials. *Carpathian J. Math.* 32 (2016), no. 1, 71–77.
- [2] M. Ayman-Mursaleen, Md. Heshamuddin, N. Rao, B. K. Sinha and A. K. Yadav, Hermite polynomials linking Szász-Durrmeyer operators. *Comput. Appl. Math.* 43 (2024), no. 4, Paper No. 223, 17 pp.
- [3] N. Raza, M. Kumar and M. Mursaleen, Approximation operators explored through the lens of degenerate Hermite polynomials. *Numer. Algorithms* 100 (2025), no. 3, 1179–1198.
- [4] N. Rao, A. K. Yadav, M. Mursaleen, B. K. Sinha and N. K. Jha, Szász-Beta operators via Hermite polynomial. *J. King Saud Univ. Sci.* 36 (2024), no. 4, Article No. 103120, 6 pp.
- [5] R. P. Gupta and G. C. Jain, A generalized Hermite distribution and its properties. *SIAM J. Appl. Math.* 27 (1974), 359–363.
- [6] P. Appell and J. Kampé de Fériet, *Fonctions Hypergéométriques et Hypersphériques: Polynômes d'Hermite*. Gauthier-Villars, Paris, 1926.
- [7] W. A. Khan, Degenerate Hermite-Bernoulli numbers and polynomials of the second kind, *Prespace-time J.* 7 (2016), no. 9, 1297–1305.



Hypergeometric Sum/Integral Transformations: Bridging Gauge Theories and Spin Models

Mustafa Mullahasanoglu

Boğaziçi University, Department of Physics, Istanbul, Türkiye
e-mail: mustafa.mullahasanoglu@std.bogazici.edu.tr

Abstract

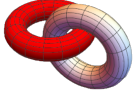
This talk explores the profound and unifying role of hypergeometric sum/integral transformations in modern mathematical physics. We discuss how identities associated with the A_n and BC_n root systems act as a powerful mathematical structure between distinct areas of physics [1]. We demonstrate that a range of mathematical techniques, such as Bailey lemma [2] and asymptotic properties [3], significantly broaden the conceptual and analytical framework available for the study of physics.

We demonstrate how these transformation formulas provide equalities between the supersymmetric indices of dual gauge theories and encode the integrability and symmetry properties of two-dimensional lattice models in statistical mechanics. By bridging gauge theories and the Yang-Baxter equation (YBE), which is called gauge/YBE correspondence [4–7], we highlight how the language of advanced special functions and their associated sum/integral identities provides a deep, unifying framework.

Keywords: Hypergeometric functions, transformation formulas, gauge theories, integrability.

References:

- [1] A. P. Kels and M. Yamazaki, Elliptic hypergeometric sum/integral transformations and supersymmetric lens index. SIGMA 14 (2018), 013.
- [2] I. Gahramanov, B. Keskin, D. Kosva and M. Mullahasanoglu, On Bailey pairs for $\mathcal{N} = 2$ supersymmetric gauge theories on $S_b^3/\mathbb{Z}_r$. JHEP 03 (2023), 169.
- [3] M. Mullahasanoglu, A. M. T. Yetkin and R. Yumusak, Ordinary limits of the hyperbolic hypergeometric integral identities. J. Phys. Conf. Ser. 3152 (2025) no. 1, 012030.
- [4] D. N. Bozkurt, I. Gahramanov and M. Mullahasanoglu, Lens partition function, pentagon identity, and star-triangle relation. Phys. Rev. D 103 (2021) no. 12, 126013.
- [5] M. Mullahasanoglu, The star-square relation and the generalized star-triangle relation from 3d supersymmetric dualities I. Eur. Phys. J. Plus 139 (2024) no. 7, 643.
- [6] E. Catak and M. Mullahasanoglu, Decorating the gauge/YBE correspondence. Eur. Phys. J. C 84 (2024) no. 9, 962.
- [7] E. Catak and M. Mullahasanoglu, Flipping relation as a reduced star-star relation, Eur. Phys. J. C 85 (2025) no.10, 1176.



Generalized Weinstein-Sobolev-Gevrey Spaces

Nahed Krir

*University of Gabès, Faculty of Sciences of Gabès, Department of Mathematics, Gabès, Tunisia
e-mail: nahed.krir.fsg@gmail.com*

Abstract

We introduce the generalized Weinstein-Sobolev-Gevrey spaces $\mathcal{H}_{a,\sigma}^{s,\nu,n}(\mathbb{R}_+^{d+1})$, defined via the generalized Weinstein transform. We establish their basic properties, including completeness, continuous embeddings, and duality. Using Saitoh's theory of reproducing kernels, we solve an extremal function problem: for a bounded linear operator $\mathcal{L} : \mathcal{H}_{a,\sigma}^{s,\nu,n} \rightarrow \mathcal{H}$, we find the unique function $f_{r,h}^*$ minimizing

$$r\|f\|_{\mathcal{H}_{a,\sigma}^{s,\nu,n}}^2 + \|h - \mathcal{L}f\|_{\mathcal{H}}^2.$$

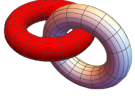
We give an explicit representation of $f_{r,h}^*$ in terms of the reproducing kernel of a modified space and provide sharp pointwise estimates. Concrete examples, including the identity and multiplier operators, are discussed in detail.

These results, published in [1], extend the theory of the CFBT to the study of extremal functions in generalized Gevrey classes, with potential applications to regularization of inverse problems.

Keywords: Sobolev-Gevrey spaces, extremal functions, reproducing kernels.

References:

- [1] H. Ben Mohamed, M. M. Chaffar and N. Krir, Extremal functions on the spaces $\mathcal{H}_{a,\sigma}^{\alpha,n}(\mathbb{R}_+^{d+1})$. *Filomat* 40 (2026), no. 5, 1667–1684.



Linear Canonical Fourier-Bessel Gabor Transform and Extremal Functions

Nahed Krir

*University of Gabès, Faculty of Sciences of Gabès, Department of Mathematics, Gabès, Tunisia
e-mail: nahed.krir.fsg@gmail.com*

Abstract

This talk presents a complete harmonic analysis framework for the canonical Fourier-Bessel transform (CFBT), a generalization of the classical Fourier-Bessel transform that unifies several integral transforms arising in optics and signal processing. The CFBT $\mathcal{F}_\nu^{\mathbf{m}}$ is parametrized by a matrix $\mathbf{m} \in SL(2, \mathbb{R})$ and is intrinsically linked to a quadratic deformation of the Bessel operator.

A key contribution is the factorization

$$e^{i(\nu+1)\frac{\pi}{2}\operatorname{sgn}(b)}\mathcal{F}_\nu^{\mathbf{m}} = \mathbf{L}_{\frac{d}{b}} \circ \mathbf{D}_b \circ \mathcal{F}_\nu \circ \mathbf{L}_{\frac{a}{b}},$$

where $\mathbf{L}_a$ and $\mathbf{D}_a$ are chirp multiplication and dilation operators, respectively, and $\mathcal{F}_\nu$ is the classical Fourier-Bessel transform. This factorization allows us to derive systematically the fundamental properties of the CFBT: Riemann–Lebesgue lemma, inversion formula, Plancherel theorem, Paley-Wiener theorem, and a range of uncertainty principles including Heisenberg, Hardy, Nash, Carlson, and Miyachi-type inequalities.

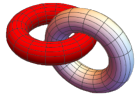
As a first application, we introduce the continuous canonical Bessel-Gabor transform $\mathcal{G}_g^{\mathbf{m}}$, which combines time–frequency analysis with the CFBT structure. We prove its boundedness on L^2 , establish Plancherel and inversion formulae, and derive a Heisenberg-type uncertainty principle. Using reproducing kernel theory, we study the problem of approximate concentration and show that if the energy of $\mathcal{G}_g^{\mathbf{m}}f$ is concentrated in a small set, then f cannot be arbitrarily localized. Furthermore, we characterize the image space $\mathcal{G}_g^{\mathbf{m}}(L^2)$ as a reproducing kernel Hilbert space and provide explicit pointwise bounds for the kernel.

These results, published in [1], contribute a unified theory for the CFBT and demonstrate its utility in time–frequency analysis.

Keywords: Linear canonical transform, Fourier-Bessel transform, Gabor transform, uncertainty principles.

References:

- [1] H. Ben Mohamed and N. Krir, Linear canonical Bessel Gabor transform. *Rend. Circ. Mat. Palermo* (2) 74 (2025), no. 1, Paper No. 64, 21 pp.



Oscillation Criteria for Difference Equations with Non-Monotone Deviating Arguments of Advanced Type

Nedjem Eddine Ramdani⁽¹⁾ and Sandra Pinelas⁽²⁾

⁽¹⁾ *University of Batna 2, Department of Mathematics, Batna, Algeria*
e-mail: nedjemeddine.ramdani@yahoo.com

⁽²⁾ *Academia Militar, Lisbon, Portugal*
e-mail: sandra.pinelas@gmail.com

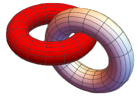
Abstract

This paper investigates the oscillation of solutions to first-order advanced difference equations with non-monotone deviating arguments. We establish two oscillation criteria based on an iterative procedure, which yield sharper conditions than existing results in the literature. Numerical examples are provided to illustrate the applicability of the obtained criteria.

Keywords: Difference equation, oscillatory criteria, characteristic equation, non-monotone advanced arguments.

References:

- [1] G. E. Chatzarakis and L. Shaikhet, Oscillation criteria for difference equations with non-monotone arguments. *Adv. Difference Equ.* 2017, Paper No. 62, 16 pp.
- [2] E. Braverman, G. E. Chatzarakis and I. P. Stavroulakis, Corrigendum: Iterative oscillation tests for difference equations with several non-monotone arguments. *J. Difference Equ. Appl.* 26 (2020), no. 6, i–v.
- [3] E. Braverman, G. E. Chatzarakis and I. P. Stavroulakis, Iterative oscillation tests for difference equations with several non-monotone arguments. *J. Difference Equ. Appl.* 21 (2015), no. 9, 854–874.
- [4] G. E. Chatzarakis and I. Jadlovská, Oscillations in deviating difference equations using an iterative technique. *J. Inequal. Appl.* 2017, Paper No. 173, 24 pp.
- [5] E. R. Attia and B. M. El-Matary, New aspects for the oscillation of first-order difference equations with deviating arguments. *Opuscula Math.* 42 (2022), no. 3, 393–413.



Estimating a Finite Population Mean Using Transformed Data in Presence of Random Non-Response

Nelson Kiprono Bii⁽¹⁾ and Christopher Ouma Onyango⁽²⁾

⁽¹⁾*Institute of Mathematical Sciences, Strathmore University, P.O Box 59857-00200, Nairobi, Kenya
e-mail: nkiprono@strathmore.edu*

⁽²⁾*Department of Statistics, Kenyatta University, P.O Box 43844-00200, Nairobi, Kenya*

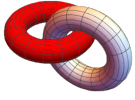
Abstract

Developing finite population estimators of parameters such as mean, variance, and asymptotic mean squared error has been one of the core objectives of sample survey theory and practice. Sample survey practitioners need to assess the properties of these estimators so that better ones can be adopted. In survey sampling, the occurrence of non-response affects inference and optimality of the estimators of finite population parameters. It introduces bias and may cause samples to deviate from the distributions obtained by the original sampling technique. To compensate for random non-response, imputation methods have been proposed by various researchers. However, the asymptotic bias and variance of the finite population mean estimators are still high under this technique. In this paper, transformation of data weighting technique is suggested. The proposed estimator is observed to be asymptotically consistent under mild assumptions. Simulated data show that the estimator proposed is much better than its rival estimators for all the different mean functions simulated.

Keywords: Mean squared error, bias, kernel regression, two-stage cluster sampling, confidence interval lengths.

References:

- [1] N. K. Bii, C. O. Onyango and J. Odhiambo, Boundary bias correction using weighting method in presence of non-response in two-stage cluster sampling. *J. Probab. Statist.* 2019 (2019), Article No. 6812795, 8 pp.
- [2] J. K. Kim and J. J. Kim, Nonresponse weighting adjustment using estimated response probability. *Canad. J. Statist.* 35 (2007), no. 4, 501–514.
- [3] F. Jay Breidt and J. D. Opsomer, Local polynomial regression estimators in survey sampling, *Ann. Statist.* 28 (2000), no. 4, 1026–1053.



A New Generalization of Touchard-Based Appell Polynomials

Neslihan Biricik Hepsisler⁽¹⁾, Bayram Çekim⁽¹⁾ and Mehmet Ali Özarslan⁽²⁾

⁽¹⁾ *Gazi University, Department of Mathematics, Ankara, Türkiye*
e-mail: neslihanbiricik@gazi.edu.tr
e-mail: bayramcekim@gazi.edu.tr

⁽²⁾ *Eastern Mediterranean University, Department of Mathematics,*
Gazimağusa, TRNC, via Mersin 10, Türkiye
e-mail: mehmetali.ozarslan@emu.edu.tr

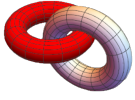
Abstract

In this study, a new generalization of the Touchard-based Appell polynomials is introduced, by the Bell and Appell polynomials. Starting from a suitably constructed generating function that builds upon the generalized Touchard polynomials, several fundamental properties are established, including a determinant representation and a recurrence relation, together with various identities obtained through the Euler derivative operator. Structural relations with the λ -Stirling numbers of the second kind are also derived, along with connections to Apostol-Bernoulli numbers of negative order and other combinatorial sequences, as well as several finite and infinite summation formulas involving the new family. In addition, 3D surface plots of certain subfamilies of this family are presented to illustrate their structural behavior.

Keywords: Touchard polynomials, Appell polynomials, determinant form, recurrence relation, Stirling numbers.

References:

- [1] E. T. Bell, Exponential polynomials. *Ann. of Math.* (2) 35 (1934), no. 2, 258–277.
- [2] P. Appell, Sur une classe de polynômes. *Ann. Sci. Éc. Norm. Supér.* 9 (1880), 119–144.
- [3] A. Y. Ceylan and Y. Simsek, Formulae for generalization of Touchard polynomials with their generating functions. *Symmetry* 17 (2025), 1126, 11 pp.
- [4] Z. Özat, M. A. Özarslan and B. Çekim, On Bell based Appell polynomials. *Turkish J. Math.* 47 (2023), no. 4, 1099–1128.



Growth of the Laguerre Transform in Terms of Modulus of Continuity

Nurbek Kakharman⁽¹⁾ and Niyaz Tokmagambetov⁽²⁾

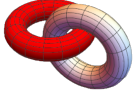
⁽¹⁾*Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan*
e-mail: n.kakharman@math.kz

⁽²⁾*Institute of Mathematics and Mathematical Modeling, Almaty, Kazakhstan*
e-mail: tokmagambetov@math.kz

Abstract

This study establishes a general inequality controlling the growth behavior of the Laguerre transform for both large and small arguments, by using a suitable class of $\mathcal{L}_\alpha^p$ -multipliers. As a consequence, a Titchmarsh theorem-type result for the Laguerre transform is obtained in the one-dimensional setting.

Keywords: Laguerre transform, modulus of smoothness, Riemann-Lebesgue lemma, Hausdorff-Young inequality, Titchmarsh's theorem.



RUC Semigroups and Their Asymptotic Limit Sets

Niyazi Anıl Gezer

TED University, Department of Mathematics, Ankara, Türkiye
e-mail: anilgezer@gmail.com

Abstract

Strongly continuous one-parameter operator semigroups, or C_0 -semigroups, play a central role in solving the abstract Cauchy problem in Banach spaces and analyzing continuous dynamical systems. The classical theory of C_0 -semigroups relies on the norm topologies of Banach spaces. In the present work, we focus on spaces where order structures dictate the underlying geometry.

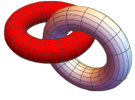
Recent literature [1] has utilized relatively uniform convergence in ordered vector spaces to introduce relatively uniformly continuous (ruc) semigroups. They effectively generalize the topological continuity of classical C_0 -semigroups. The use of convergence regulators provides a flexible, order-theoretic approach to analyze semigroup continuity at zero without depending on a standard topological norm.

In this talk, we explore the local properties of trajectories generated by these semigroups and formally introduce the relatively uniform omega (ω_{ru}) and alpha (α_{ru}) limit sets. We establish that under appropriate lattice conditions, such as the property (R) , these ru-limit sets exhibit invariance under the semigroup action and satisfy certain closure properties.

Keywords: C_0 -semigroups, relatively uniform convergence, asymptotic behavior.

References:

- [1] E. Emelyanov, N. Erkuşun-Özcan and S. Gorokhova, Relatively uniformly continuous semigroups on ordered vector spaces. J. Math. Anal. Appl. 560 (2026), no. 2, Paper No. 130556, 13 pp.



On the Image of the Limit q -Stancu Operator

Ovgu Gurel

Recep Tayyip Erdogan University, Department of Mathematics, Rize, Türkiye
e-mail: ovgu.gurelyilmaz@erdogan.edu.tr

Abstract

The limit q -Stancu operator $S_{\infty}^{q,\alpha}$, $0 < q < 1$ and $\alpha > 0$, arises as the limit form of the q -Stancu polynomials introduced by Nowak [2] and constitutes a significant generalization of the classical Bernstein operators.

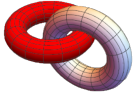
Our main goal is to understand how analytic properties of functions are reflected in their images through $S_{\infty}^{q,\alpha}$. It is proved that the image of every continuous function on $[0, 1]$ can be extended analytically to a certain disk in the complex plane. It is also shown that having more derivatives at the point $x = 1$ leads to a larger domain of analyticity. In particular, if a function is infinitely differentiable at $x = 1$, then its image is an entire function. Furthermore, growth estimates for the image functions are obtained. Finally, it is established that the image of any transcendental entire function grows strictly slower than the function itself unlike the case of polynomials where $S_{\infty}^{q,\alpha}$ owns the degree preserving property.

This talk is based on a joint work with Sofiya Ostrovska and Mehmet Turan from Atılım University, Türkiye.

Keywords: q -Bernstein operator, limit q -Stancu operator, growth estimate.

References:

- [1] F. Baraner, A new aspect of q -Stancu operators, MSc Thesis, 2025.
- [2] G. Nowak, Approximation properties for generalized q -Bernstein polynomials. J. Math. Anal. Appl. 350 (2009), no. 1, 50–55.
- [3] G. Nowak, A de Casteljau algorithm for q -Bernstein-Stancu polynomials. Abstr. Appl. Anal. 2011, Art. ID 609431, 13 pp.
- [4] S. Ostrovska, The q -versions of the Bernstein operator: from mere analogies to further developments, Results Math. 69 (2016), no. 3-4, 275–295.
- [5] D. D. Stancu, Approximation of functions by a new class of linear polynomial operators, Rev. Roumaine Math. Pures Appl. 13 (1968), 1173–1194.



A Class of Generalized Gegenbauer Polynomials

Özge Ada⁽¹⁾ and Esra Erkuş-Duman⁽²⁾

⁽¹⁾ *Gazi University, Graduate School of Natural and Applied Sciences,
Department of Mathematics, Ankara, Türkiye
e-mail: ozge.aadaa@gmail.com*

⁽²⁾ *Gazi University, Faculty of Science, Department of Mathematics, Ankara, Türkiye
e-mail: eduman@gazi.edu.tr*

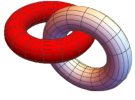
Abstract

A derivative-based extension of the classical Gegenbauer polynomials is proposed by introducing an additional independent variable, leading to a new family of bivariate polynomials. The construction is examined through its algebraic and analytic characteristics, and explicit formulas together with the corresponding generating function are established. Several identities, including recurrence formulas, differential and integral relations, are obtained, providing effective tools for investigating the proposed polynomial family. Connections with Stirling numbers and power sums are also derived, yielding a variety of combinatorial identities. Finally, the generating function is extended to a more general form, from which further identities and consequences for the new polynomial family are deduced. The results enrich the theory of generalized Gegenbauer polynomials and provide a basis for further developments in the study of special polynomial sequences.

Keywords: Gegenbauer polynomials, recurrence relation, generating function, symmetry identity.

References:

- [1] E. Erkuş-Duman, A new perspective on generalized Laguerre polynomials. *Demonstr. Math.* 58 (2025), no. 1, Paper No. 20250162, 14 pp.
- [2] E. D. Rainville, *Special Functions*. The Macmillan Company, New York, 1960.
- [3] E. Erkuş-Duman and H. Ciftci, A new family of two-variable polynomials based on Hermite polynomials. *Math. Slovaca* 72 (2022), no. 4, 885–898.
- [4] Ö. Ada and E. Erkuş-Duman, A derivative-based extension of Gegenbauer polynomials in two variables. *Nuclear Phys. B* 1025 (2026), Paper No. 117393, 9 pp.



Dynamics of Oxygen-Phytoplankton Interactions: Understanding Global Warming through the Allee Effect

Pinar Baydemir⁽¹⁾, Huseyin Merdan⁽²⁾ and William F. Fagan⁽³⁾

⁽¹⁾ *TOBB University of Economics and Technology, Department of Mathematics, Ankara, Türkiye*
e-mail: pinarbaydemir@etu.edu.tr

⁽²⁾ *TOBB University of Economics and Technology, Department of Mathematics, Ankara, Türkiye*
e-mail: merdan@etu.edu.tr

⁽³⁾ *University of Maryland, Department of Biology, College Park, MD, USA*
e-mail: bfagan@umd.edu

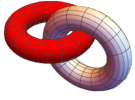
Abstract

In this presentation, we investigate the dynamic behavior of an oxygen-phytoplankton interaction model [1] incorporating a strong Allee effect. To characterize how the system evolves, we employ both the continuous-time differential equation framework and its discrete-time counterpart [2]. Initially, the local stability conditions of the biologically relevant equilibrium points—extinction, Allee threshold, and positive coexistence—are determined for both systems. The prerequisites for the presence of specific bifurcations are next studied. We demonstrate that the continuous system undergoes a transcritical bifurcation driven by the Allee threshold, acting as a tipping point between survival and collapse. Conversely, the discrete-time system exhibits flip bifurcations leading to chaotic dynamics as the time step size varies [3]. We have observed that the Allee equilibrium functions as a critical separatrix; trajectories below this threshold collapse to extinction, while those above may converge to coexistence. To rigorously quantify the vulnerability of the system beyond standard stability analysis, we utilize the resilience index to estimate the probability of persistence based on the basin of attraction, and the first passage time function to detect critical slowing down near these tipping points. Our numerical simulations reveal a “ghost of attractor” phenomenon [4], indicating that the system may exhibit low resilience by lingering in a vulnerable depletion zone even before a total collapse. Ultimately, this study serves as a theoretical foundation for understanding the ecological consequences of global warming, demonstrating how environmental stress can push seemingly stable ecosystems across critical thresholds toward irreversible collapse.

Keywords: Allee effect, global warming, stability analysis, tipping point, bifurcation analysis, ghost of attractor, ecological resilience.

References:

- [1] Y. Sekerci and S. Petrovskii, Mathematical modelling of spatiotemporal dynamics of oxygen in a plankton system. *Math. Model. Nat. Phenom.* 10 (2015), 96–114.
- [2] P. Baydemir and H. Merdan, Bifurcation analysis, chaos control, and FAST approach for the complex dynamics of a discrete-time predator–prey system with a weak Allee effect. *Chaos, Solitons Fractals* 196 (2025), Paper No. 116317, 13 pp.
- [3] Y. A. Kuznetsov, *Elements of Applied Bifurcation Theory*. 3rd Edition, Springer, New York, 2004.
- [4] A. Hastings, K. C. Abbott, K. Cuddington, T. Francis, G. Gellner, Y. C. Lai, A. Morozov, S. Petrovskii, K. Scranton and M. L. Zeeman, Transient phenomena in ecology. *Science* 361 (2018), eaat6412.



On a q -Deformed Pythagorean Triples

Raymond Calvin Ochieng⁽¹⁾, Maurice Oduor Owino⁽²⁾ and Vitalis Onyango-Otieno⁽³⁾

⁽¹⁾*Strathmore University, Strathmore Institute of Mathematical Sciences, Nairobi, Kenya*
e-mail: rochieng@strathmore.edu

⁽²⁾*University of Kabianga, Department of Mathematics and Actuarial Science, Kericho, Kenya*
e-mail: moduor@kabianga.ac.ke

⁽³⁾*Strathmore University, Strathmore Institute of Mathematical Sciences, Nairobi, Kenya*
e-mail: vonyango@strathmore.edu

Abstract

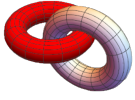
Recent work by Mathevet, Morier-Genoud, and Ovsienko (2026) introduced a novel q -deformation of the classical Pythagoras equation, $\mathcal{A}(q)^2 + q\mathcal{B}(q)^2 = \mathcal{C}(q)\mathcal{C}^*(q)$, constructing an infinite family of polynomial solution triples systematically indexed by standard Pythagorean triples through a q -deformed action of the modular group $PSL(2, \mathbb{Z})$. While this dynamic framework successfully generates standard solutions, it leaves alternative 'exotic' solution orbits—discovered via numerical experimentation—algebraically isolated and unclassified.

In this paper, we propose an alternative, static classification paradigm by embedding the q -deformed Pythagoras equation directly into the commutative quadratic ring extension $\mathcal{R} = \mathbb{Z}[q][\sqrt{-q}]$. By reinterpreting the left-hand polynomial combination as an algebraic norm, the task of finding valid polynomial triples is reframed as a problem of characterizing principal ideals invariant under a reciprocal involution operator. Utilizing a generalized polynomial form of the Brahmagupta–Fibonacci identity, we establish the exact structural and degree-bound constraints required to satisfy the necessary monic, positive-coefficient, and self-reciprocal boundary conditions. This ring-theoretic approach provides a rigorous algebraic mechanism that accounts for the existence of non-factorable exotic solutions, mapping them to distinct polynomial orbits outside the scope of the standard modular tree. Finally, we discuss how this framework can be leveraged to address the open conjecture on coefficient unimodality using complex root distribution on the unit circle.

Keywords: q -Deformed Pythagoras equation, ring extensions, quadratic rings.

References:

- [1] F. Acquistapace, F. Broglia, J. Fernando and J. Ruiz, On the Pythagoras numbers of real analytic surfaces. *Ann. Sci. École Norm. Sup. (4)* 38 (2005), no. 5, 751–772.
- [2] S. Amari and A. Ohara, Geometry of q -exponential family of probability distributions. *Entropy* 13 (2011), no. 6, 1170–1185.
- [3] H. Mathevet, S. Morier-Genoud and V. Ovsienko, Quantizing Pythagorean triples. arXiv:2602.20536, 2026.
- [4] J. M. R. Caballero, A q -analog of Jacobi's two squares formula and its applications. *Hardy-Ramanujan J.* 45 (2022), 84–87.
- [5] J. Zhang and T.-K. L. Wong, λ -deformation: a canonical framework for statistical manifolds of constant curvature. *Entropy* 24 (2022), no. 2, Paper No. 193, 26 pp.



Gamma Type Operators that Preserve Logarithmic Functions

Reyhan Canatan İlbey

Baskent University, Kahramankazan Vocational School, Ankara, Türkiye
e-mail: rilbey@baskent.edu.tr

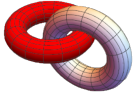
Abstract

In this talk, Classical Gamma-type operators that preserve logarithmic functions is introduced utilizing the methods outlined in [1]. Additionally, we examine the convergence properties of these operators and a quantitative estimate of the approximation error is derived in relation to the modulus of continuity.

Keywords: Gamma-type operators, logarithm preservation.

References:

- [1] L. Angeloni, D. Costarelli and C. Darielli, A new class of positive linear operators preserving logarithmic functions. arXiv preprint, arXiv:2602.06168. (2026).



Tail Asymptotics for Products of Generalized Gamma Random Variables

Ričardas Kamarauskas, Aurimas Slabovas and Jonas Šiaulyš

Vilnius University, Institute of Mathematics, Vilnius, Lithuania

e-mail: ricardas.kamarauskas@mif.stud.vu.lt

e-mail: aurimas.slabovas@mif.stud.vu.lt

e-mail: jonas.siaulyš@mif.vu.lt

Abstract

Products of random variables arise in reliability theory, wireless communications, risk modelling, and related areas of applied probability. Motivated by our previous work on products of Weibull random variables [1], we consider the product of independent generalized gamma random variables with non-identical parameters. More precisely, let

$$\Pi_n = \prod_{k=1}^n \xi_k, \quad \xi_k \sim GG(a_k, d_k, p_k), \quad k = 1, \dots, n,$$

where $GG(a, d, p)$ denotes the generalized gamma distribution with density

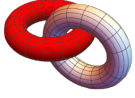
$$f(x) = \frac{(p/a^d)x^{d-1}e^{-(x/a)^p}}{\Gamma(d/p)}, \quad x > 0.$$

We derive a sharp asymptotic formula for the survival probability $\mathbb{P}(\Pi_n > x)$ as $x \rightarrow \infty$. The proof relies on a transformation analogous to that used in [2], the integral representation of the product tail, and the multidimensional Laplace method; see, for instance, [3].

Keywords: Generalized gamma distribution, product of random variables, Laplace method, tail distribution function.

References:

- [1] R. Kamarauskas, A. Slabovas and J. Šiaulyš, Asymptotic expansions for products of Weibull random variables. *Mathematics* 14 (2026), no. 4, Article No. 736, 23 pp.
- [2] J. Pitman and M. Z. Rácz, Beta-gamma tail asymptotics. *Electron. Commun. Probab.* 20 (2015), no. 84, 7 pp.
- [3] R. Wong, *Asymptotic Approximations of Integrals*. Academic Press, Boston, 1989.



Comparative Approximation Analysis of Generalized Neural Network Operators on Damped Time Series

Seda Karateke

*Department of Software Engineering, Istanbul Atlas University, 34408 İstanbul, Türkiye
e-mail: seda.karateke@atlas.edu.tr*

Abstract

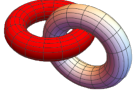
Neural network operators form a constructive framework for quantitative approximation and numerical signal modeling. Motivated by sigmoidal neural network approximation theory [1], fractional neural network approximation [2], recent infinite-domain ordinary and fractional quasi-interpolation schemes [3], and generalized neural network operator constructions [4], this study compares three denominator-free neural network operators on noisy damped time-series data: the sampling-type S_n operator, the Kantorovich-type K_n operator, and the quadrature-type Q_n operator. The theoretical framework is based on modulus-of-continuity-type Jackson estimates and on the fractional approximation viewpoint involving left and right Caputo fractional derivatives. In the computational setting, S_n uses pointwise samples, K_n incorporates local averaging through integral means, and Q_n replaces continuous integration by weighted numerical quadrature. Numerical validation is performed on a 60-month damped harmonic oscillator signal with additive Gaussian noise. The first 48 months are used for hyperparameter selection, and the final 12 months are reserved for blind testing. The S_n operator achieves an MAE of 0.7787 and an R^2 value of 0.9518, while K_n yields an MAE of 0.8142 and an R^2 value of 0.9389. The Simpson-based Q_n operator closely reproduces the Kantorovich performance while reducing the cost of repeated integral evaluations.

These findings indicate that denominator-free neural network operators provide an efficient and theoretically grounded approximation strategy for noisy damped time-series signals.

Keywords: Neural network operators, damped time series, approximation metrics.

References:

- [1] D. Costarelli and R. Spigler, Approximation results for neural network operators activated by sigmoidal functions. *Neural Networks*, 44 (2013), 101–106.
- [2] G. A. Anastassiou, Fractional neural network approximation. *Comput. Math. Appl.* 64 (2012), no. 6, 1655–1676.
- [3] G. A. Anastassiou, Smooth symmetrized and perturbed hyperbolic tangent real and complex, ordinary and fractional neural network approximations over infinite domains. *Ann. Commun. Math.* 7 (2024), no. 4, 401–429.
- [4] N. Ö. İpekci, S. Karateke and İ. Büyükyazıcı, Approximation by generalized neural network operators (*accepted for publication*).



An Investigation of a Special Type-2 Fuzzy Metric

Şehla Eminoglu⁽¹⁾, Tülay Yazır^(1,2) and Tahir Khaniyev⁽³⁾

⁽¹⁾ *Ankara Yıldırım Beyazıt University, Department of Mathematics, Ankara, Türkiye*
e-mail: sehla.eminoglu@aybu.edu.tr

⁽²⁾ *Karadeniz Technical University, Department of Mathematics, Trabzon, Türkiye*
e-mail: tkesemen@ktu.edu.tr

⁽³⁾ *TOBB University of Economics and Technology, Dept. of Industrial Engineering, Ankara, Türkiye*
e-mail: tahirkhaniyev@etu.edu.tr

Abstract

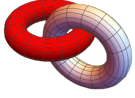
Many real-world problems contain not only uncertainty but also additional ambiguity about the reliability of that uncertainty. Classical mathematical models based on precise distance definitions often encounter significant limitations in practical applications due to measurement errors, perceptual inconsistencies, and unfavorable environmental conditions. Consequently, the reconsideration of fundamental notions such as proximity, similarity, and distance in uncertain environments has emerged as an important research topic from both theoretical and practical perspectives. In this regard, Type-2 fuzzy numbers, originally introduced by Zadeh [1] and subsequently developed in depth by Mendel [2], provide a powerful framework for modeling higher-order uncertainty. Unlike Type-1 fuzzy sets, where uncertainty is represented solely through membership grades, Type-2 fuzzy sets incorporate uncertainty directly into the membership functions themselves. This additional degree of freedom enables a more comprehensive representation of imprecise and ambiguous information. The concept of α -cuts, which plays a central role in the analysis of Type-1 fuzzy numbers, has been generalized to (α, β) -cuts for Type-2 fuzzy numbers. This extension has provided an effective analytical tool for investigating the structural and mathematical properties of Type-2 fuzzy numbers. In particular, approaches based on (α, β) -sections have bridged a significant gap in the literature by facilitating the systematic study of Type-2 fuzzy quantities within a rigorous mathematical framework. On the other hand, Kaleva and Seikkala [3] extended the classical notion of a metric by introducing a distance function whose values are non-negative Type-1 fuzzy numbers.

Motivated by the increasing need to model higher-order uncertainty, the primary objective of the present study is to incorporate the second-order uncertainty represented by Type-2 fuzzy sets into the metric framework originally proposed by Kaleva and Seikkala. To this end, a Type-2 fuzzy metric obtained by especially the absolute value function is constructed explicitly through (α, β) -sections, thereby providing a natural and systematic extension of classical fuzzy arithmetic to the Type-2 setting.

Keywords: Type-2 fuzzy metric, (α, β) -level sets, second-order uncertainty.

References:

- [1] L. A. Zadeh, Fuzzy sets. Information and Control 8 (1965), no. 3, 338–353.
- [2] J. M. Mendel, Advances in type-2 fuzzy sets and systems. Inform. Sci. 177 (2007), no. 1, 84–110.
- [3] O. Kaleva and S. Seikkala, On fuzzy metric spaces. Fuzzy Sets and Systems 12 (1984), no. 3, 215–229.



Donoho-Stark Uncertainty Principles in the Laguerre Hypergroup Setting

Soumeiya Hamem^(1,2) and Selma Negzaoui^(1,2)

⁽¹⁾ *University of Monastir, Preparatory Institute of Engineering Studies of Monastir, Monastir, Tunisia*
e-mail: soumeiya.hamem@gmail.com

⁽²⁾ *Université de Tunis El Manar, Faculté des Sciences de Tunis, Laboratoire d'Analyse Mathématique et Applications LR11ES11, Tunis, Tunisia*
e-mail: selma.negzaoui@fst.utm.tn

Abstract

The Donoho-Stark uncertainty principle quantifies the impossibility of a nonzero function and its Fourier transform being simultaneously highly concentrated on small sets. This work extends this principle to the Laguerre hypergroup, a non-Euclidean harmonic analysis framework.

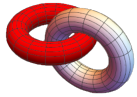
We first prove Donoho-Stark theorems for the Fourier-Laguerre transform, establishing quantitative bounds relating the measures of concentration sets in the spatial and frequency domains.

We then extend these results to the Continuous Laguerre Wavelet Transform (CLWT), obtaining new concentration inequalities in the time-scale domain.

Our main contribution is the introduction of localization operators associated with the CLWT. For functions that are approximately localized with respect to two such operators, we prove a Donoho-Stark-type inequality that provides a lower bound on the product of the measures of the corresponding concentration sets.

These results extend classical uncertainty principles beyond Euclidean settings and offer new tools for time-scale signal analysis on hypergroups.

Keywords: Donoho-Stark uncertainty principle, Fourier-Laguerre transform, continuous Laguerre wavelet transform, concentration operators, localization operators, Laguerre hypergroup.



Degenerate Sheffer F -Polynomials and Their Properties

Semra Kuş

Kırşehir Ahi Evran University, Mucur Vocational High School, Kırşehir, Türkiye
e-mail: skus@ahievran.edu.tr

Abstract

The Sheffer F -polynomial sequence $\{s_n(x)\}_{n \geq 0}$ is defined by $s_0(x) = \text{const} \neq 0$ and

$$Q(\partial_F) s_n(x) = F_n s_{n-1}(x), \quad (n \geq 0),$$

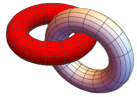
where $Q(\partial_F)$ is delta F -operator in [1].

In this study, we introduce the degenerate Sheffer F -polynomials by means of a combination of F -calculus and degenerate sequences. Furthermore, utilizing the F -umbral calculus and the F -derivative, we derive various properties of these polynomials and establish their relations with certain specific fibonomial-based degenerate polynomials.

Keywords: Degenerate polynomials, Sheffer sequences, Fibonomial calculus.

References:

- [1] E. Krot, An introduction to finite fibonomial calculus. Cent. Eur. J. Math. 2 (2004), no. 5, 754–766.
- [2] S. Kuş, N. Tuglu and T. Kim, Bernoulli F -polynomials and Fibo-Bernoulli matrices. Adv. Difference Equ. 2019, Paper No. 145, 16 pp.
- [3] S. Roman, The Umbral Calculus. Academic Press, New York, 1984.
- [4] L. Carlitz, A degenerate Staudt-Clausen theorem. Arch. Math. (Basel) 7 (1956), 28–33.
- [5] L. Carlitz, Degenerate Stirling, Bernoulli and Eulerian numbers. Utilitas Math. 15 (1979), 51–88.
- [6] M. Ozvatan, Generalized Golden-Fibonacci Calculus and Applications. Master's thesis, Izmir Institute of Technology, 2018.
- [7] D. S. Kim and T. Kim, Degenerate Sheffer sequence and λ -Sheffer sequence. J. Math. Anal. Appl. 493 (2021), no. 1, Paper No. 124521, 21 pp.
- [8] M. Riyasat, M. Haneef and S. Khan, Some properties of degenerate Sheffer sequences based on algebraic approach. Indian J. Pure Appl. Math. 56 (2025), no. 1, 414–431.
- [9] S. A. Wani, Two-iterated degenerate Appell polynomials: properties and applications. Arab J. Basic Appl. Sci. 31 (2024), no. 1, 83–92.
- [10] S. Kuş and N. Tuglu, On the degenerate Bernoulli's F -polynomials. In: Proceedings of the Karaelmas International Science and Engineering Symposium (KISES 2024), Zonguldak, Türkiye, 2024.
- [11] A. Kim, Zeros of the degenerate Bernoulli-Fibonacci polynomials. J. Appl. Pure Math. 6 (2024), no. 5, 231–240.
- [12] O. Dışkaya, Degenerate Bernoulli-Fibonacci and Euler-Fibonacci polynomials. Complex Anal. Synerg. 11 (2025), no. 4, Paper No. 19, 13 pp.



Generalization of Jakimovski-Leviatan-Păltănea Operators Based on Special Polynomials

Kadir Kanat⁽¹⁾, Melek Sofyaloğlu Aksoy⁽¹⁾ and Sena Nur Sancak⁽²⁾

⁽¹⁾ *Ankara Hacı Bayram Veli University, Polath Faculty of Science and Arts, Department of Mathematics, Ankara, Türkiye*

e-mail: kadir.kanat@hbv.edu.tr

e-mail: melek.sofyalioglu@hbv.edu.tr

⁽²⁾ *Ankara Hacı Bayram Veli University, Institute of Graduate Studies, Department of Mathematics, Ankara, Türkiye*

e-mail: sena.sancak@ogr.hbv.edu.tr

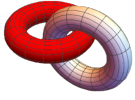
Abstract

In this presentation, a new family of Jakimovski–Leviatan–Păltănea type positive linear operators, generated via a general framework incorporating special polynomials, is introduced and investigated on the unbounded interval $[0, \infty)$. By embedding the exponential moment properties of this general framework, convergence properties are obtained. To form the basis of the theoretical analysis, explicit representations for the moments and central moments are derived using generating function techniques. While uniform convergence properties are established via a Korovkin-type theorem, the rate of convergence is computed with the aid of the modulus of continuity and the Lipschitz class. To better elucidate these convergence characteristics, the theoretical results are supported by structured comparison tables. Furthermore, weighted approximation results are examined in suitable weighted spaces, and a Voronovskaya-type asymptotic theorem is proved to characterize the asymptotic behavior of the approximation error.

Keywords: Jakimovski–Leviatan–Păltănea operators, Korovkin-type theorem, Voronovskaya-type theorem .

References:

- [1] A. Jakimovski and D. Leviatan, Generalized Szász operators for the approximation in the infinite interval. *Mathematica (Cluj)* 11(34) (1969), 97–103.
- [2] R. Păltănea, *Approximation Theory Using Positive Linear Operators*. Birkhäuser, Boston, 2004.
- [3] A. Gadzhiev, The convergence problem for a sequence of positive linear operators on unbounded sets, and theorems analogous to that of P. P. Korovkin. *Dokl. Akad. Nauk SSSR*, 218 (1974), 1001–1004.
- [4] I. Yüksel and N. Ispir, Weighted approximation by a certain family of summation integral-type operators. *Comput. Math. Appl.* 52 (2006), no. 10-11, 1463–1470.
- [5] K. J. Ansari, M. A. Salman, M. Mursaleen and A. H. H. Al-Abied, On Jakimovski-Leviatan-Paltanea approximating operators involving Boas-Buck-type polynomials. *J. King Saud Univ. Sci.* 32 (2020), no. 7, 3018–3025.



On Almost A -Statistical Convergence for Double Sequences and Korovkin-Type Approximation

Sevda Yıldız⁽¹⁾ and Selin Çınar⁽²⁾

⁽¹⁾*Sinop University, Department of Mathematics, Sinop, Türkiye*
e-mail: sevdaorhan@sinop.edu.tr

⁽²⁾*Sinop University, Department of Mathematics, Sinop, Türkiye*
e-mail: scinar@sinop.edu.tr

Abstract

In approximation theory, classical convergence is sometimes insufficient to analyze the behavior of positive linear operators, necessitating the use of generalized summability methods. Motivated by recent developments for single sequences, the primary aim of this study is to first introduce the concept of almost A -statistical convergence for double sequences. Specifically, for a subset $F \subseteq \mathbb{N}_0^2$ and an almost regular four-dimensional matrix $A = (a_{nmij})$, we define the almost A -density of F as

$$\delta_A^{2,a}(F) := P - \lim_{p,q \rightarrow \infty} \frac{1}{pq} \sum_{n=s}^{s+p-1} \sum_{m=t}^{t+q-1} \sum_{(i,j) \in F} a_{nmij}$$

uniformly in s and t . Consequently, a double sequence $x = (x_{ij})$ is said to be almost A -statistically convergent to a limit L if for every $\epsilon > 0$, the following condition holds:

$$\delta_A^{2,a}(\{(i,j) \in \mathbb{N}_0^2 : |x_{ij} - L| \geq \epsilon\}) = 0.$$

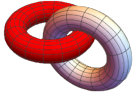
Following this new mathematical construction, we establish a Korovkin-type approximation theorem for double sequences of positive linear operators acting on the space of continuous functions of two variables. To demonstrate the significance and effectiveness of our theoretical results, we construct an illustrative example of a sequence of positive linear operators that fails to converge in the classical or ordinary statistical sense but converges under our newly proposed method. Finally, the theoretical developments are supported by MATLAB-based numerical experiments. Graphical representations are provided to illustrate the convergence behavior of the constructed example, substantiating the mathematical claims with computational evidence.

Keywords: Almost A -summability, double sequences, Korovkin-type theorem, positive linear operators.

Acknowledgements: The first author has been supported by Scientific and Technological Research Council of Turkey (TUBITAK)/ 2224-B-Grant Program for Participation in Scientific Meetings within the Country. The authors thanks to TUBITAK for their supports.

References:

- [1] K. Demirci, F. Dirik and S. Yıldız, Almost A -statistical convergence and approximation theorems. *Filomat* 39 (2025), no. 23, 8117–8128.
- [2] P. P. Korovkin, On convergence of linear positive operators in the space of continuous functions (in Russian). *Doklady Akad. Nauk SSSR (N.S.)* 90 (1953), 961–964.
- [3] G. G. Lorentz, A contribution to the theory of divergent sequences. *Acta Math.* 80 (1948), 167–190.
- [4] F. Móricz and B. E. Rhoades, Almost convergence of double sequences and strong regularity of summability matrices. *Math. Proc. Cambridge Philos. Soc.* 104 (1988), no. 2, 283–294.
- [5] M. Mursaleen and E. Savaş, Almost regular matrices for double sequences. *Studia Sci. Math. Hungar.* 40 (2003), no. 1-2, 205–212.



On Cesàro Submethods and Statistical Convergence in Neutrosophic Normed Spaces

Şeyda Sezgek⁽¹⁾ and İlhan Dağadur⁽²⁾

⁽¹⁾ *Mersin University, Department of Mathematics, Mersin, Türkiye*
e-mail: seydasezgek@mersin.edu.tr

⁽²⁾ *Mersin University, Department of Mathematics, Mersin, Türkiye*
e-mail: idagadur@mersin.edu.tr

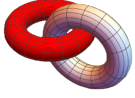
Abstract

Neutrosophic normed space (NNS) was defined and some important properties of NNS and statistical convergence with respect to NNS were investigated by [2]. In this study, we expand on this work by examining further properties of the Cesàro submethods associated with a strictly increasing sequence $\lambda = (\lambda_n)$ of positive integers defined by [1]. Namely, we examine further the inclusion properties of these submethods and their relationship to statistical convergence in neutrosophic settings. A counterexample is also constructed to show that the inclusion between these sequence classes is strict in general. In addition, we introduce the concepts of C_λ -completeness and D_λ -completeness. We prove that every neutrosophic normed space is complete with respect to these convergence methods.

Keywords: Neutrosophic normed space, statistical convergence, density, deferred convergence, Cauchy sequence.

References:

- [1] J. A. Osikiewicz, Equivalence results for Cesàro submethods. *Analysis (Munich)* 20 (2000), no. 1, 35–43.
- [2] M. Kirisci and N. Şimsek, Neutrosophic normed spaces and statistical convergence. *J. Anal.* 28 (2020), no. 4, 1059–1073.



Dynamical Analysis of a Leslie-Gower Predator-Prey Model with Fear Effect and Prey Refuge

Seyma Bilazeroğlu⁽¹⁾ and Beyza Acun⁽²⁾

⁽¹⁾ *Çankaya University, Department of Computer Science, Ankara, Türkiye*
e-mail: sbilazeroğlu@cankaya.edu.tr

⁽²⁾ *Çankaya University, Department of Mathematics, M.Sc. Student, Ankara, Türkiye*
e-mail: beyzo-talha@hotmail.com

Abstract

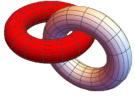
In this paper, we investigate a ratio-dependent predator–prey model incorporating both fear effect and prey refuge mechanisms. The model is formulated within the Leslie-Gower framework [1], where predator growth depends on the ratio between prey and predator populations. The fear effect is introduced through a nonlinear reduction in the prey reproduction rate, while the refuge mechanism assumes that a constant portion of the prey population is protected from predation.

We first establish the positivity and invariance of solutions in the biologically meaningful region. Then, the existence and uniqueness of positive equilibrium points are obtained for all considered submodels. Local stability of the equilibria is analyzed by means of the Jacobian matrix and eigenvalue criteria. In particular, conditions ensuring stable node and stable spiral behavior are derived explicitly in terms of the model parameters. Furthermore, global dynamical properties of the full system are investigated. Sufficient conditions guaranteeing global asymptotic stability of the positive equilibrium are established. The obtained results show that the combined effects of fear and refuge significantly influence the stability structure and long-term dynamics of the predator-prey interaction.

Keywords: Predator-prey model, ratio-dependent, fear effect, refuge, global stability.

References:

- [1] P. H. Leslie, Some further notes on the use of matrices in population mathematics. *Biometrika* 35 (1948), no. 3-4, 213–245.



Some Applications of Power Series Statistical Convergence

Şeyma Çelik⁽¹⁾ and Dilek Söylemez⁽²⁾

⁽¹⁾ *Selcuk University, Department of Mathematics, Konya, Türkiye*
e-mail: seymacelik216@gmail.com

⁽²⁾ *Selcuk University, Department of Mathematics, Konya, Türkiye*
e-mail: dsozden@gmail.com

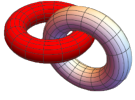
Abstract

This study investigates the approximation properties of modified Baskakov operators via power series statistical convergence. By analyzing these operators within weighted function spaces, we explore their convergence characteristics on unbounded intervals and establish quantitative estimates for the rate of convergence using the weighted modulus of continuity. Our findings highlight the effectiveness of power series statistical methods in extending the applicability of linear positive operators. The theoretical results are supported by illustrative examples and graphical illustrations to demonstrate the efficiency of the operators.

Keywords: Power series statistical convergence, weighted Korovkin-type theorem, rate of convergence.

References:

- [1] O. Agratini, Linear operators that preserve some test functions. *Int. J. Math. Math. Sci.* 2006, Art. ID 94136, 11 pp.
- [2] Ö. G. Atlıhan, T. Yurdakadim, and E. Taş, A new approach to the approximation by positive linear operators in weighted spaces. *Ukrainian Math. J.* 74 (2023), no. 11, 1649–1657.
- [3] V. A. Baskakov, An example of a sequence of linear positive operators in the space of continuous functions. *Doklady Akademii Nauk SSSR* 113 (1957), 249–25.
- [4] N. İspir, On modified Baskakov operators on weighted spaces. *Turkish J. Math.* 25 (2001), no. 3, 355–365.
- [5] L. Rempulska and K. Tomczak, Approximation by certain linear operators preserving x^2 . *Turkish J. Math.* 33 (2009), no. 3, 273–281.
- [6] M. Ünver and C. Orhan, Statistical convergence with respect to power series methods and applications to approximation theory. *Numer. Funct. Anal. Optim.* 40 (2019), no. 5, 535–547.



A Topological Characterization of Fragments via Zero Sets

Sezer Bolat⁽¹⁾ and Nazife Erkuşun-Özcan⁽²⁾

⁽¹⁾*Hacettepe University, Department of Mathematics, Ankara, Türkiye
e-mail: sezer.bolat314@gmail.com*

⁽²⁾*Hacettepe University, Department of Mathematics, Ankara, Türkiye
e-mail: erkursun.ozcan@hacettepe.edu.tr*

Abstract

Freudenthal's spectral theorem is one of the most important statements in the theory of vector lattices [1]. It fundamentally highlights the importance of investigating the fragments of positive elements within these spaces. Recent developments have begun to bridge this lattice-theoretic framework with topology, establishing initial connections between the fragment structures of continuous functions and their associated zero sets [2].

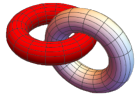
In this work, we deepen this connection by providing a direct topological characterization of fragments within the space $C[0, 1]$. Specifically, we prove that a continuous function $f \in C[0, 1]$ admits a proper fragment if and only if its zero set $Z(f)$, endowed with the subspace topology, contains a nontrivial clopen subset. Moreover, motivated by recent advancements in the complexification of vector lattices [3], we extend our topological approach to the complex function space $C[0, 1] + iC[0, 1]$. By analyzing the zero components of the real and imaginary parts, we establish explicit topological criteria for the existence of proper complex fragments.

This talk is based on an ongoing joint work with N. Erkuşun-Özcan.

Keywords: Vector lattices, continuous function spaces, fragment structure, zero set, clopen set, complexification.

References:

- [1] C. D. Aliprantis and O. Burkinshaw, Positive Operators. Springer, Dordrecht, 2006.
- [2] S. Bolat, N. Erkuşun-Özcan and N. A. Gezer, On fragments on lattice normed vector lattices. Results Math. 78 (2023), no. 6, Paper No. 245, 13 pp.
- [3] N. Dzhusoeva, J. Huang, M. Pliev and F. Sukochev, Lateral order on complex vector lattices and narrow operators. Math. Nachr. 296 (2023), no. 11, 5157–5170.



Multiple Binomial-Type Kantorovich Operators and Their Approximation Properties

Mehmet Ali Özarslan⁽¹⁾, Bayram Çekim⁽²⁾ and Sezin Çit⁽²⁾

⁽¹⁾ *Eastern Mediterranean University, Department of Mathematics,
Gazimagusa, TRNC Mersin 10, Türkiye
e-mail: mehmetali.ozarslan@emu.edu.tr*

⁽²⁾ *Gazi University, Department of Mathematics, Ankara, Türkiye
e-mail: bayramcekim@gazi.edu.tr
e-mail: sezincit@gazi.edu.tr*

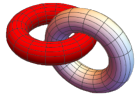
Abstract

Binomial-type polynomial sequences, first introduced by Popoviciu [1], provide a natural setting for constructing positive linear operators. Building on this, Sablonnière [2] introduced the Bernstein-Sheffer operators and investigated their approximation properties on $C[0, 1]$. In this work, we introduce multiple Bernstein-Kantorovich operators as a Kantorovich-type extension [3] of the multiple binomial-type operators recently studied in [4]. These operators are constructed via multiple binomial-type polynomial sequences defined through the exponential generating function $e^{xH(t_1, t_2)}$, extending the classical single-index framework to a multi-index setting. We establish the moments and central moments of the operators and prove uniform convergence via the Korovkin theorem. Quantitative estimates are obtained using the classical modulus of continuity, the second-order modulus of smoothness, and Peetre's κ -functional. Approximation rates for Lipschitz-type function classes are also derived. In particular, the linear choice H yields the classical Bernstein-Kantorovich operator of degree $2n$, achieving an improved convergence rate by a factor of $1/\sqrt{2}$ compared to the classical operator of degree n . Finally, for the Bell-type generating function H , the second central moment of the multiple operator is asymptotically half that of the corresponding single-index operator, independently of any additional parameters.

Keywords: Multiple Bernstein-Kantorovich operators, modulus of continuity, Sheffer polynomials, Peetre's κ -functional, Bell numbers.

References:

- [1] T. Popoviciu, Remarques sur les polyômes binomiaux. Bull. Soc. Ştiinţ. Cluj. 6 (1931), 146–148.
- [2] P. Sablonnière, Positive Bernstein-Sheffer operators. J. Approx. Theory 83 (1995), no. 3, 330–341.
- [3] L. V. Kantorovich, Sur certains développements suivant les polyômes de la forme de S. Bernstein. C. R. Acad. Sci. URSS. 31 (1930), 477–479.
- [4] M. A. Özarslan, B. Çekim and S. Çit, Multiple binomial-type operators and their approximation properties. J. Math. 2026 (2026), 7852651, 14 pp.



On Cesàro Summability of Fourier-Bessel series and Its Consequences

Sagar Saini⁽¹⁾ and Shailesh Kumar Srivastava⁽²⁾

⁽¹⁾*Department of Mathematics, Sardar Vallabhbhai National Institute of Technology, Surat, India
e-mail: sagarmathwork@gmail.com*

⁽²⁾*Department of Mathematics, Sardar Vallabhbhai National Institute of Technology, Surat, India
e-mail: shaileshiitr2010@gmail.com*

Abstract

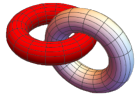
In this paper, we study the Cesàro summability of order δ , (C, δ) , $\delta > -1$, on the Fourier-Bessel series, and using the n^{th} -Cesàro mean of order ' δ ', we approximate functions with a jump discontinuity, for which the classical Fourier-Bessel series partial sums exhibit Gibbs phenomena near the point of discontinuity and show ringing behaviour at points away from the discontinuity [1, 2]. Inspired by the classical theory of Cesàro summability for trigonometric Fourier series [3, 4], we formulate the (C, δ) summability of the Fourier-Bessel series and develop the corresponding theory and results. In particular, an explicit kernel representation analogous to the trigonometric Fourier series is introduced, and its basic properties are studied.

In the numerical study, the approximation is illustrated using a benchmark function, a step function with a jump at $x = 1/2$. A systematic comparison is shown by initially fixing the number of terms (N) in the n^{th} (C, δ) -mean and varying the summability factor δ , and then by fixing δ while varying N . The obtained results show that for sufficiently large N , Cesàro means mitigate Gibbs phenomena near discontinuities, and reduce oscillations which is present in partial sums. Moreover, we observe that as δ increases with fixed N , the graph of the Cesàro mean becomes noticeably smoother but simultaneously deviates from the true value of the function. Therefore, small positive values of δ are more effective than larger ones. Overall, these results show that the (C, δ) , $\delta > -1$ means of Fourier-Bessel series provide an effective method for approximating functions, especially when the functions have jump discontinuities.

Keywords: Fourier-Bessel series, Cesàro summability, Gibbs phenomenon.

References:

- [1] G. N. Watson, A Treatise on the Theory of Bessel Functions. 2nd ed., Cambridge University Press, Cambridge, 1944.
- [2] T. H. Fay and P. H. Kloppers, The Gibbs phenomenon for Fourier-Bessel series. Internat. J. Math. Ed. Sci. Tech. 34 (2003), no. 2, 199–217.
- [3] H. Dutta and B. E. Rhoades, Current Topics in Summability Theory and Applications. Springer, Singapore, 2016.
- [4] A. Zygmund, Trigonometric Series. Vols. I-II, 3rd ed., Cambridge University Press, Cambridge, 2002.



DFT-Based Computational Prediction of Hypothetical Cubic C1b SrMgSi and SrMgSn Half-Heusler Phases

S. Djellab^(1,2) and B. Ouchene^(2,4), L. Tairi^(2,3), S. Ghemid⁽²⁾ and H. Meradji⁽²⁾

⁽¹⁾ *Department of Physics, Faculty of Sciences, 20 August 1955 University, Skikda, Algeria
e-mail:s.djellab@univ-skikda.dz, djellab_sihem@yahoo.fr*

⁽²⁾ *LPR Laboratory, Department of Physics, Faculty of Sciences, Badji Mokhtar University, Annaba, Algeria*

⁽³⁾ *Research Center in Industrial Technologies CRTI, P.O. Box 64, Cheraga, Algiers, Algeria*

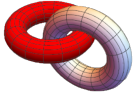
⁽⁴⁾ *LNCTS Laboratory, Department of Physics, Faculty of Sciences, Badji Mokhtar University, Algeria*

Abstract

The search for new functional materials with suitable electronic, optical, and thermoelectric properties is of great interest for future energy and optoelectronic technologies. In this context, a first-principles computational investigation is carried out on the hypothetical cubic C1b Half-Heusler compounds SrMgSi and SrMgSn.

Using density functional theory (DFT), the study examines their structural, electronic, optical, mechanical, and thermoelectric properties. The electronic structure is analyzed within both PBE-GGA and mBJ-GGA approximations, and the thermoelectric properties are evaluated using the BoltzTraP code. The main objective is to assess the potential of these materials for optoelectronic and thermoelectric applications.

Keywords: DFT, C1b half-Heusler phases, SrMgSi, SrMgSn, thermoelectric properties.



On The Analytical Properties of The Telephone Polynomials

Duriye Korkmaz-Duzgun^(1,2) and Simanur Başaran⁽³⁾

⁽¹⁾ *Ankara Hacı Bayram Veli University, Department of Mathematics, Ankara, Türkiye*

⁽²⁾ *Kafkas University, Department of Mathematics, Kars, Türkiye*
e-mail: dryekorkmaz@gmail.com

⁽³⁾ *Eskişehir Technical University, Department of Mathematics, Eskişehir, Türkiye*
e-mail: simanur372@gmail.com

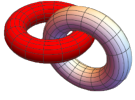
Abstract

In this study, several analytical properties of the telephone polynomials will be investigated. First, a Mehler-type formula that provides a new bilinear generating relation for this polynomial family and different types of generating function relations will be established to reveal the structural properties of the telephone polynomials. In addition, a mixed-type recurrence relation and a determinantal representation for the telephone polynomials will be derived, and their asymptotic behavior will be investigated. The obtained results are expected to contribute to the theory of special functions and orthogonal polynomials and to provide a basis for further studies in this area.

Keywords: Telephone polynomials, generating function, Mehler-type formula.

References:

- [1] K. Prasad and M. Kumari, The telephone polynomials: an Appell-type orthogonal polynomials connecting Hermite-Laguerre polynomials. *Nuclear Physics B* (2025), Paper No. 11719, 13 pp.
- [2] R. M. Hafez, H. M. Ahmed, O. M. Alqubori, A. K. Amin and W. M. Abd-Elhameed, Efficient spectral Galerkin and collocation approaches using telephone polynomials for solving some models of differential equations with convergence analysis. *Mathematics* 13 (2025), no. 6, Paper No. 918, 29 pp.



On Rough Topological Spaces

Sourav Mandal

*National Institute of Technology Durgapur, Department of Mathematics, Durgapur, India
e-mails: souravmandalnbu@gmail.com and sourav.mandal@nitdgp.ac.in*

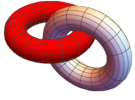
Abstract

The notion of a rough family has been recently introduced in [1, 2] to explore rough ideal convergence in topological spaces. This study primarily aims to introduce novel topological concepts, such as the rough interior and rough closure of a set from the perspective of a rough family, leading to the definition of rough open sets and rough closed sets. As a consequence, we generate a new topology in a given topological space, referred to as a rough topology. We characterize this rough topology and examine its relationship with the existing topology. Furthermore, we extend the notion of homeomorphism to rough homeomorphism and construct several examples that demonstrate that two non-homeomorphic spaces can be roughly homeomorphic under certain roughness. Our findings diverge from some previous results, generate novel properties.

Keywords: Rough family, rough open set, rough closed set, rough topology, rough homeomorphism.

References:

- [1] P. Leonetti, Rough Families, Cluster points, and cores. *J. Convex Anal.* 32 (2025), no. 4, 1083–1090.
- [2] P. Leonetti, On rough ideal convergence. *arXiv preprint arXiv:2601.13805*, 2026.



An Operator Version of the Korovkin Theorem for Modified Bernstein-Kantorovich Operators and Simultaneous Approximation

Şule Yüksel Güngör⁽¹⁾, Mehmet Ali Özarslan⁽²⁾ and Bayram Çekim⁽³⁾

⁽¹⁾ *Gazi University, Department of Mathematics, Ankara, Türkiye*
e-mail: sulegungor@gazi.edu.tr

⁽²⁾ *Eastern Mediterranean University, Department of Mathematics,
Gazimagusa, TRNC Mersin 10, Türkiye*
e-mail: mehmetali.ozarslan@emu.edu.tr

⁽³⁾ *Gazi University, Department of Mathematics, Ankara, Türkiye*
e-mail: bayramcekim@gazi.edu.tr

Abstract

In this talk, we introduce a generalized family of positive linear operators that unifies the modified Bernstein-Kantorovich operators studied by Özarslan and Duman [3] with the operator-theoretic Korovkin framework developed by Popa [4], Kiran Kumar and Vinaya [2], and Dorai [1]. Our operators map continuous functions on a closed interval into continuous functions on a compact Hausdorff space, and incorporate a shape parameter that influences both the limit operator and the rate of approximation.

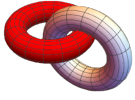
We establish a uniform convergence theorem, showing that the convergence of the operator sequence to a weighted composition operator is equivalent to the uniform convergence of an underlying sequence of continuous functions. This theorem extends and incorporates several known results from the literature as special cases.

Our main contribution is a simultaneous approximation theorem, showing that not only the operators but also their derivatives of any order converge uniformly to the corresponding derivatives of the limit operator, and we characterize this simultaneous convergence as equivalent to the uniform convergence of the underlying sequence of continuous functions. The proof relies on finite difference operators, the Faà di Bruno formula, and partial Bell polynomials. We also establish quantitative estimates for the rate of convergence for functions in Lipschitz classes.

Keywords: Bernstein-Kantorovich operators, Korovkin approximation theorem, simultaneous approximation.

References:

- [1] A. Dorai, An operator version of the Korovkin theorem revisited. *J. Math. Anal. Appl.* 535 (2024), no. 2, Paper No. 128239, 4 pp.
- [2] V. B. Kiran Kumar and P. C. Vinaya, Operator version of Korovkin theorem; degree of convergence and application to preconditioners. *J. Math. Anal. Appl.* 523 (2023), no. 1, Paper No. 127087, 11 pp.
- [3] M. A. Özarslan and O. Duman, Smoothness properties of modified Bernstein-Kantorovich operators. *Numer. Funct. Anal. Optim.* 37 (2016), no. 1, 92–105.
- [4] D. Popa, An operator version of the Korovkin theorem. *J. Math. Anal. Appl.* 515 (2022), no. 1, Paper No. 126375, 16 pp.



Fractional-Order Mathematical Model of Genetically Predisposed Diabetes Mellitus and Graphical Representation of Model Classes

Sibel Atik⁽¹⁾ and Sümeyra Uçar⁽²⁾

⁽¹⁾ *Department of Mathematics, Balıkesir University, Çağış Campus, Balıkesir, 10145, Türkiye*
e-mail: sblatik90@gmail.com

⁽²⁾ *Department of Mathematics, Balıkesir University, Çağış Campus, Balıkesir, 10145, Türkiye*
e-mail: sumeyraucar@balikesir.edu.tr

Abstract

Diabetes is a chronic metabolic disease whose prevalence is rapidly increasing worldwide and significantly affects the quality of life of individuals. Genetic predisposition is considered an important risk factor in the development of the disease. Genetic predisposition is defined as an increased risk of developing diabetes due to an individual's inherited characteristics. Diabetes is not a disease that develops suddenly, but rather the result of changes that accumulate over many years. This shows that an individual's past health status and genetic risks have an impact on the current disease process. While mathematical models created with classical derivatives take into account the instantaneous change in the system, fractional derivatives can also model the impact of past conditions on current dynamics. This feature provides a significant advantage in terms of being able to evaluate effects such as genetic predisposition cumulatively over time.

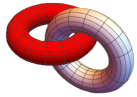
This study will employ a fractional-order mathematical model to examine the effect of genetic predisposition on the development of diabetes. In the model, genetic predisposition will be considered one of the key factors influencing the transition to prediabetes, and the effects of this process on diabetic patients and those with complicated diabetes will be investigated. Figures obtained for different values of parameters representing genetic predisposition will show the time-dependent changes in diabetes and complicated diabetes classifications.

Acknowledgement: The study is supported by Balıkesir University under the Grant no. BAP 2026/039.

Keywords: Diabetes, mathematical modeling.

References:

- [1] A. De Gaetano, T. Hardy, B. Beck, E. Abu-Raddad, P. Palumbo, J. Bue-Valleskey and N. Porksen, Mathematical models of diabetes progression. *Amer. J. Physiol. Endocrinol. Metab.* 295 (2008), no. 6, 1462-1479.
- [2] J. P. Bai, J. C. Earp and V. C. Pillai, Translational quantitative systems pharmacology in drug development: from current landscape to good practices. *AAPS J.* 21 (2019), no. 4, 72.
- [3] I. Podlubny, *Fractional Differential Equations*. Academic Press, New York, 1999.
- [4] A. Kilbas, H. M. Srivastava and J. J. Trujillo, *Theory and Applications of Fractional Differential Equations*. Vol. 204, Elsevier, 2006.
- [5] F. Evirgen, S. Uçar and N. Özdemir, Mathematical analysis and optimal control of a Caputo fractional diabetes system with parameter identification. *J. Comput. Appl. Math.* 477 (2026), Paper No. 117151, 17 pp.



Max-Product Shepard-Lidstone Operators and Their Approximation Properties

Türkan Yeliz Gökçer Ellidokuz

Bahçeşehir University, Department of Mathematics, İstanbul, Türkiye
e-mail: ylzgkr@gmail.com; yeliz.ellidokuz@bau.edu.tr

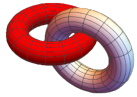
Abstract

Recently, Shepard operators have been combined with local interpolation operators such as Taylor, Bernoulli, and Lagrange polynomials to improve their approximation properties. Furthermore, nonlinear versions of these combined Shepard operators using the max-product operators [1] have been studied for the approximation of nonnegative functions [2–4]. Motivated by these results, in this presentation we consider the combined Shepard–Lidstone operators introduced in [5–7] and improve the convergence result established in [6] by showing that the convergence holds for all $\lambda \geq 1$, whereas the previous result required the condition $\lambda > 2r - 1$. We also present the max-product Shepard–Lidstone operator and establish its rate of convergence together with its approximation properties. The proposed nonlinear operator exhibits improved approximation properties compared with the classical Shepard and Shepard-Lidstone operators. In addition, we apply regular summability methods to suitably modified sequences of operators in cases where classical convergence fails. Finally, we demonstrate the theoretical findings using numerical examples.

Keywords: Max-product operators, Shepard-type operators, Lidstone Polynomials, rates of convergence.

References:

- [1] B. Bede, H. Nobuhara, M. Daňková and A. Di Nola, Approximation by pseudo-linear operators. *Fuzzy Sets and Systems* 159 (2008), no. 7, 804–820.
- [2] O. Duman, Improved approximation via hybrid Shepard-Lagrange operators: linear and nonlinear perspectives. *J. Comput. Appl. Math.* 473 (2026), Paper No. 116864, 14 pp.
- [3] O. Duman, Max-product Shepard operators based on multivariable Taylor polynomials. *Comput. Appl. Math.* 437 (2024), Paper No. 115456, 15 pp.
- [4] T. Y. Gökçer Ellidokuz, Approximation with max-product Shepard-Bernoulli operators. *J. Comput. Appl. Math.* 482 (2026), Paper No. 117313, 13 pp.
- [5] F. A. Costabile and F. Dell’Accio, Lidstone approximation on the triangle. *Appl. Numer. Math.* 52 (2005), no. 4, 339–361.
- [6] T. Cătiuaş, The combined Shepard-Lidstone univariate operator. *Tiberiu Popoviciu Itinerant Seminar of Functional Equations, Approximation and Convexity. Cluj-Napoca, May 21–25, 2003*, pp. 3–15.
- [7] R. Wu, H. Li and T. Wu, Univariate Lidstone-type multiquadric quasi-interpolants. *Comput. Appl. Math.* 39 (2020), no. 3, Paper No. 141, 19 pp.



What are the Advantages of the Granular Derivative in Solving Fuzzy Differential Equations?

Ömer Akin⁽¹⁾ and Tahir Ceylan⁽²⁾

⁽¹⁾ *TOBB Economics and Technology University, Department of Mathematics, Ankara, Türkiye*
e-mail: omerakin@etu.edu.tr

⁽²⁾ *Sinop University, Department of Mathematics, Sinop, Türkiye*
e-mail: tceylan@sinop.edu.tr

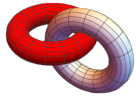
Abstract

In this talk, we use recently proposed concepts of horizontal membership function (HMF) and granular derivative based on granular difference to introduce and study the equivalence relation between granular differential equations and a class of fuzzy differential equations (FDEs). It is well known that in the study of FDEs fuzzy interval arithmetic and differentiability concept are very important. But some restrictions imposed by the approaches which are based on these concepts make it difficult to solve FDEs. Some of restrictions associated to previous approaches—Hukuhara differentiability, generalized Hukuhara differentiability- are the existence of unnatural behavior in the modeling phenomenon, the existence of multi-solutions with different geometric presentations, the diameter of the fuzzy function be monotonic, the result of each of the four basic operations performed on fuzzy numbers is not a fuzzy number etc. So thanks to the relative-distance-measure (RDM) fuzzy interval arithmetic based on RDM variables enable direct introducing uncertain interval or fuzzy variable-values in usual mathematical formulas together with crisp values. Moreover, we compare the proposed approach with other ones based on other derivatives using some examples. In addition, we discuss the efficiency and effectiveness of the proposed approach in detail.

Keywords: Fuzzy differential equations (FDEs), granular differentiability, horizontal membership functions, relative-distance-measure (RDM).

References:

- [1] M. Najariyan, N. Pariz and H. Vu, Fuzzy linear singular differential equations under granular differentiability concept. *Fuzzy Sets and Systems* 429 (2022), 169–187.
- [2] M. Mazandarani, N. Pariz and A. V. Kamyad, Granular differentiability of fuzzy-number-valued functions. *IEEE Trans. Fuzzy Syst.* 26 (2018), no. 1, 310–323.
- [3] A. Khastan and J. J. Nieto, A boundary value problem for second order fuzzy differential equations. *Nonlinear Anal.* 72 (2010), no. 9-10, 3583–3593.
- [4] A. Piegat and M. Landowski, Horizontal membership function and examples of its application. *Int. J. Fuzzy Syst.* 17 (2015), no. 1, 22–30.
- [5] M. L. Puri and D. A. Ralescu, Differentials of fuzzy functions, *J. Math. Anal. Appl.* 91 (1983), no. 2, 552–558.



Approximation Approach in State Estimation Problems for Dynamical Control Systems Under Uncertainty

Tatiana F. Filippova

*Krasovskii Institute of Mathematics and Mechanics, Department of Optimal Control,
Ural Branch of Russian Academy of Sciences, Ekaterinburg, Russia
e-mail: ftf@imm.uran.ru*

Abstract

The control problems for set-valued trajectories of a nonlinear dynamic system described by differential equations with nonlinearity and uncertainty are studied. It is assumed that only the bounding sets for uncertain variables are known and any additional statistical information is not available. Approaches are described for solving the optimal control problem for such systems, including the problem of the fastest transfer of the system state inside a given terminal set.

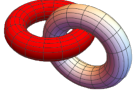
The research develops previously proposed theoretical schemes and improves basic approaches considered earlier in [1,2]. Numerical simulations related to the proposed techniques and based on new state estimation ideas and on new algorithms are included to illustrate the main results.

The work was performed as part of research conducted in the Ural Mathematical Center with the financial support of the Ministry of Science and Higher Education of the Russian Federation (Agreement number: 075-02-2026-737).

Keywords: Nonlinear dynamics, uncertainty, ellipsoidal calculus.

References:

- [1] T. F. Filippova, On two approaches to the solution of set-valued state estimation problem for control systems under uncertainty. *J. Math. Sci. (N.Y.)* 295 (2025), no. 3, 316–324.
- [2] T. F. Filippova, HJB-Inequalities in estimating reachable sets of a control system under uncertainty. *Ural Math. J.* 8 (2022), no. 1, 34–42.



An Approximate Formula for the Expected Value of Residual Waiting Time Process under Heavy-Tailed Distributions

Tülay Yazır^(1,2), Aşlı Bektaş Kamışlık⁽³⁾ and Tahir Khaniyev^(4,5)

⁽¹⁾ *Karadeniz Technical University, Department of Mathematics, Trabzon, Türkiye*

⁽²⁾ *Ankara Yıldırım Beyazıt University, Department of Mathematics, Ankara, Türkiye*
e-mail: tkesemen@ktu.edu.tr

⁽³⁾ *Recep Tayyip Erdogan University, Department of Mathematics, Rize, Türkiye*
e-mail: asli.bektas@erdogan.edu.tr

⁽⁴⁾ *TOBB Economics and Technology University, Department of Industrial Engineering, Ankara, Türkiye*

⁽⁵⁾ *Azerbaijan State University of Economics, The Center of Digital Economics, Baku, Azerbaijan*
e-mail: tahirkhaniyev@etu.edu.tr

Abstract

Let $W(x) = S_{\nu(x)} - x$ be the residual waiting time of a renewal process, $\nu(x) = \inf\{n \geq 1 : S_n > x\}$ and let $R(x) = \mathbb{E}[W(x)]$. For non-negative i.i.d. interarrival times with distribution function F , survival function $\bar{F}$, and finite mean μ , the identity $R(x) = \mu U(x) - x$ transfers the bounds for renewal-function $U(x)$ directly to $R(x)$. Chadjiconstantinidis [1] obtained exponential-type bounds involving the root $\kappa(x) > 0$ defined by $\int_0^x e^{\kappa(x)y} dF(y) = 1$, which yield

$$\mu \frac{1 - e^{-x\kappa(x)}}{\bar{F}(x)} - x \leq R(x) \leq \mu \frac{1 - e^{-2x\kappa(x)}}{\bar{F}(x)} - x.$$

To facilitate the numerical computation of $\kappa(x)$, we construct an analytical initial bracket using the truncated moments $m_j(x) = \int_0^x y^j dF(y)$, $j = 1, 2$:

$$\frac{1}{x} \log \left(1 + \frac{x\bar{F}(x)}{m_1(x)} \right) \leq \kappa(x) \leq \frac{-m_1(x) + \sqrt{m_1^2(x) + 2m_2(x)\bar{F}(x)}}{m_2(x)}.$$

If $[a_n(x), b_n(x)]$ is the interval obtained after n bisection steps, its midpoint is defined by $\hat{\kappa}_n(x) = (a_n(x) + b_n(x))/2$. Substituting $\hat{\kappa}_n(x)$ into the mean of the two bounds gives the following approximation for expected value of residual waiting time process $R(x)$:

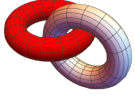
$$R_{\text{bapp},n}(x) = \frac{\mu}{2\bar{F}(x)} \left[2 - e^{-x\hat{\kappa}_n(x)} - e^{-2x\hat{\kappa}_n(x)} \right] - x.$$

The method is illustrated for the heavy-tailed Pareto/Lomax distribution with shape $\alpha = 2$ and scale $\lambda = 1$, and compared with the numerical values reported in [1]. Numerical comparisons based on the values reported in [1] demonstrate the applicability of the method in a heavy-tailed setting.

Keywords: residual waiting time, renewal process, exponential-type bounds, bisection method, heavy-tailed distributions.

References:

- [1] S. Chadjiconstantinidis, Exponential and Pareto-type bounds for the renewal function and the excess lifetime of a renewal process, *Statistics* 58(6) (2024), 1443–1462.



New Enclosures for Bessel Functions

Ulrich Reif

*Technical University of Darmstadt, Department of Mathematics, Darmstadt, Germany
e-mail:reif@mathematik.tu-darmstadt.de*

Abstract

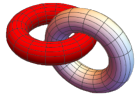
We present a new paradigm for deriving enclosures for Bessel functions J_ν and Y_ν of the first and second kind for real arguments and orders. It is based on a hitherto unknown monotonicity property of a differential operator related to the modulus function $M_\nu := \sqrt{J_\nu^2 + Y_\nu^2}$. When applied to the asymptotic expansions of the modulus M_ν and the phase $\Theta_\nu := \arctan Y_\nu/J_\nu$, we obtain significantly tighter enclosures than the known ones. Moreover, former results for the asymptotic expansion of M_ν apply only if the number of summands exceeds the order ν and results for the asymptotic expansion of Θ_ν apply only if $\nu < 3/2$. By contrast, our findings apply to any number of summands and arbitrary orders $\nu \geq 0$, thus extending the covered range substantially.

Further, we introduce the most simple, yet powerful one-term approximation

$$M_\nu(x) \approx \sqrt{\frac{2}{\pi \sqrt{x^2 - \nu^2 + 1/4}}}$$

of the modulus. We discuss its convergence properties, specify enclosures for itself and the resulting approximation of the phase Θ_ν , and conclude with some remarks concerning convergent bounds on the location of the zeros of J_ν and Y_ν .

Keywords: Bessel function, asymptotic expansion, enclosure.



On a New Class of Multivariable k -Gottlieb Polynomials and Their Analytical Properties

Yahya Çin⁽¹⁾ and Nejlâ Özmen⁽²⁾

⁽¹⁾ *Düzce University, Department of Mathematics, Düzce, Türkiye*
e-mail: yahyacin2525@gmail.com

⁽²⁾ *Düzce University, Department of Mathematics, Düzce, Türkiye*
e-mail: nejlaozmen@duzce.edu.tr; nejlaozmen06@gmail.com

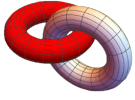
Abstract

In this paper, we introduce a new class of multivariable k -Gottlieb polynomials as a k -analogue of the classical multivariable Gottlieb polynomials and investigate their fundamental properties. These polynomials are defined via an appropriate generating function, which serves as the main tool for deriving explicit representations. By employing analytic and operational techniques, we establish several key results, including generating function identities and a Mellin transform formula. Furthermore, we obtain various integral representations that provide additional insight into their structural properties. The results presented here extend the theory of multivariable Gottlieb polynomials and contribute to the development of k -special functions.

Keywords: Multivariable k -Gottlieb polynomials, k -analysis, generating function, integral representation.

References:

- [1] N. Özmen and Y. Çin, Exploration of k -hypergeometric polynomials and their mathematical implications. *Filomat* 39 (2025), no. 26, 9329–9345.
- [2] G. Rahman, S. Nisar Mubeen and K. Sooppy, On generalized k -fractional derivative operator. *AIMS Math.* 5 (2020), no. 3, 1936–1945.
- [3] M. A. Khan and M. Akhlaq, Some new generating functions for Gottlieb polynomials of several variables. *Internat. Trans. Appl. Sci.* 1 (2009), no. 4, 567–570.
- [4] E. Erkuş-Duman, N. Özmen, On the Gottlieb Polynomials in several variables. *Facta Univ. Ser. Math. Inform.* 34 (2019), no. 5, 927–940.
- [5] R. Diaz and E. Pariguan, On hypergeometric functions and Pochhammer k -symbol. *Divulg. Mat.* 15 (2007), no. 2, 179–192.
- [6] Y. Çin and N. Özmen, Generalized k -Cesàro polynomials. *Dolomites Res. Notes Approx.* 18 (2025), no. 2, 64–78.



A New Generalization of Truncated-Type Polynomials and Their Properties

Yasin Ünsal⁽¹⁾, Can Kızılates⁽²⁾ and Esra Erkuş-Duman⁽³⁾

⁽¹⁾ *Zonguldak Bülent Ecevit University, Department of Mathematics, Zonguldak, Türkiye*
e-mail: yasnunsal066@gmail.com

⁽²⁾ *Zonguldak Bülent Ecevit University, Department of Mathematics, Zonguldak, Türkiye*
e-mail: cankizilates@gmail.com

⁽³⁾ *Gazi University, Department of Mathematics, Ankara, Türkiye*
e-mail: eerkusduman@gmail.com

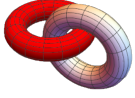
Abstract

Truncated versions of classical special polynomials have been receiving growing attention in recent years, largely due to their connections with generating functions, combinatorial identities, and hypergeometric-type structures. Motivated by this interest, the present work introduces a new generalization of truncated-type polynomials built on a modification of the classical Frobenius–Euler generating function. The construction relies on a finite truncation applied to the exponential part of the generating function, which naturally gives rise to a flexible framework that includes several well-known polynomial families as special cases. Within this setting, we study a number of fundamental properties of the introduced polynomials: generating functions, recurrence relations, differential identities, and representation formulas involving Stirling numbers of the second kind. We also touch on connections with hypergeometric functions and take a closer look at how the zeros of these polynomials behave in the complex plane.

Keywords: Frobenius-Euler polynomials, truncated polynomials, generating functions, Stirling numbers, zeros of polynomials.

References:

- [1] K. Shiratani and S. Yamamoto, On a p -adic interpolation function for the Euler numbers and its derivatives. *Mem. Fac. Sci. Kyushu Univ. Ser. A* 39 (1985), no. 1, 113–125.
- [2] T. Komatsu and C. Pita-Ruiz, Truncated Euler polynomials. *Math. Slovaca* 68 (2018), no. 3, 527–536.
- [3] A. Hassen and H. D. Nguyen, Hypergeometric Bernoulli polynomials and Appell sequences. *Int. J. Number Theory* 4 (2008), no. 5, 767–774.



A Deformation-Adaptive First-Order Optimization Framework Using Jackson q -Gradients

Otmane Filali Razzouki⁽¹⁾, Zakaria El Allali⁽²⁾, Anas Charoud⁽²⁾ and Abdelaziz Chetouani⁽¹⁾

⁽¹⁾ *ENCG, Mohammed First University in Oujda, Morocco*
e-mails: otmane.filalirazzouki.d24@ump.ac.ma
e-mails: a.chetouani@ump.ac.ma

⁽²⁾ *Team of MSC, Department of Mathematics, FPN, Mohammed First University in Oujda, Morocco*
e-mails: z.elallali@ump.ma
e-mails: a.charroud@ump.ac.ma

Abstract

We introduce Variable Jackson q -Gradient Descent (VJqGD), a first-order optimization method [1], inspired by recent advances in accelerated first-order methods for non-convex optimization. The proposed approach replaces the classical gradient with a Jackson q -gradient and progressively anneals the deformation parameter according to

$$q_k = 1 + (q_0 - 1)e^{-k/\tau}.$$

This mechanism enables a smooth transition from an exploratory non-local optimization regime toward classical gradient descent behavior as $q_k \rightarrow 1$, consistent with prior observations regarding the exploratory capabilities of q -gradient-based methods [2]. We further establish theoretical guarantees showing that, under standard smoothness assumptions and for $q > 1$ sufficiently close to 1, the Jackson q -gradient defines a valid descent direction on bounded sets, in line with modern analyses of first-order optimization methods [3].

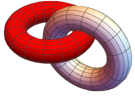
The proposed method is evaluated on a diverse benchmark suite including Ackley, Rastrigin, Levy, Rosenbrock, Michalewicz, SumSquares, and EggHolder functions. Comparisons with Gradient Descent, Adam, RMSprop, and Adagrad are conducted using statistical tests and Dolan-Moré performance profiles. Experimental results demonstrate that VJqGD achieves competitive convergence and improved robustness on multimodal and ill-conditioned optimization landscapes, while also revealing limitations on highly deceptive functions such as Michalewicz.

Keywords: Gradient descent, q -calculus, q -derivative, Armijo rule, adaptive optimization.

Acknowledgements: This article is based upon work from COST Action 24122 mSPACE, supported by COST (European Cooperation in Science and Technology), www.cost.eu.

References:

- [1] O. F. Razzouki, A. Charroud, Z. El Allali, A. Chetouani and N. Aslimani, A survey of advanced gradient methods in machine learning. In Proc. 7th International Conference on Advanced Communication Technologies and Networking (CommNet), Rabat, Morocco, 2024, pp. 1–7.
- [2] Y. Yang and B. Yang, An efficient non-negative least mean squares algorithm based on q -gradient for system identification. Digital Signal Processing 148 (2024), 104438, 9 pp.
- [3] T. Rotaru, Exact Performance Analysis of Fundamental First-Order Optimization Methods. Ph.D. Dissertation, UCLouvain, 2026.



Investigation of Hyperbolic Cahn-Hilliard Equations with Dynamic Boundary Conditions

Zehra Nur Öztürk⁽¹⁾ and Sema Yayla⁽²⁾

⁽¹⁾*Eskişehir Technical University, Department of Information Systems and Technologies,
Eskişehir, Türkiye
e-mail: znozturk@eskisehir.edu.tr*

⁽²⁾*Hacettepe University, Department of Mathematics, Ankara, Türkiye
e-mail: semasimsek@hacettepe.edu.tr*

Abstract

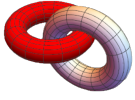
Cahn-Hilliard equations were introduced to model phase separation processes in binary alloy-like materials [1]. In this work, we investigate the hyperbolic Cahn-Hilliard equation without a viscosity term under dynamic boundary conditions. We modify the boundary conditions in order to estimate certain terms arising in the analysis of the problem. Then, we define weak solutions for the problem and establish the existence of these solutions through the Faedo-Galerkin approximation method.

This work is supported by the Scientific and Technological Research Council of Türkiye (TÜBİTAK) through the ARDEB 3501 Career Development Program (Project No. 123F453).

Keywords: Hyperbolic Cahn-Hilliard equation, dynamic boundary conditions, Faedo-Galerkin method, weak solutions, energy methods.

References:

- [1] J. W. Cahn and J. E. Hilliard, Free energy of a nonuniform system. I. Interfacial free energy. J. Chem. Phys. 28 (1958), no. 2, 258–267.
- [2] C. G. Gal, A Cahn-Hilliard model in bounded domains with permeable walls. Math. Methods Appl. Sci. 29 (2006), no. 17, 2009–2036.
- [3] R. Racke and S. Zheng, The Cahn-Hilliard equation with dynamic boundary conditions. Adv. Differential Equations 8 (2003), no. 1, 83–110.
- [4] K. Binder and P. Fratzl, Spinodal Decomposition, Phase Transformations in Materials. Wiley-VCH, 2001.
- [5] P. Galenko and V. Lebedev, Non-equilibrium effects in spinodal decomposition of a binary system. Phys. Lett. A 372 (2008), no. 7, 985–989.
- [6] C. G. Gal and M. Grasselli, Singular limit of viscous Cahn-Hilliard equations with memory and dynamic boundary conditions. Discrete Contin. Dyn. Syst. Ser. B 18 (2013), no. 6, 1581–1610.
- [7] C. Cavaterra, M. Grasselli and H. Wu, Non-isothermal viscous Cahn-Hilliard equation with inertial term and dynamic boundary conditions. Commun. Pure Appl. Anal. 13 (2014), no. 5, 1855–1890.
- [8] P. Knopf and C. Liu, On second-order and fourth-order elliptic systems consisting of bulk and surface PDEs: Well-posedness, regularity theory and eigenvalue problems. Interfaces Free Bound. 23 (2021), 507–533.



On Appell Polynomials Associated with k -Hypergeometric Functions and Their Properties

Zeynep Özat⁽¹⁾, Bayram Çekim⁽¹⁾ and Mehmet Ali Özarslan⁽²⁾

⁽¹⁾*Gazi University, Department of Mathematics, Ankara, Türkiye*
e-mail: zeynepozat95@gmail.com
e-mail: bayramcekim@gazi.edu.tr

⁽²⁾*Eastern Mediterranean University, Department of Mathematics,
Gazimagusa, TRNC Mersin 10, Türkiye*
e-mail: mehmetali.ozarslan@emu.edu.tr

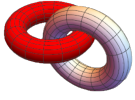
Abstract

In recent years, hypergeometric functions have been used to introduce new generalizations of certain particular polynomial families. For example, confluent Appell polynomials have been defined by considering the confluent hypergeometric function, and various properties of these polynomials have been investigated [1]. Furthermore, the literature also mentions the k -analogue of hypergeometric functions [2, 3]. The purpose of this study is to analyze the k -analogue of Appell polynomials and introduce k -confluent Appell polynomials. A few of these polynomials corresponding characteristic characteristics are investigated, including its recurrence relation, explicit representation, summation formulas, determinant representation, and various properties. When we focus on the special case of k , these polynomials reduce to the confluent Appell polynomials. Additionally, subfamilies containing special cases of the determining function and their properties are given.

Keywords: Confluent Appell polynomials, hypergeometric functions, generating function.

References:

- [1] M. A. Özarslan and B. Çekim, Confluent Appell polynomials. J. Comput. Appl. Math. 424 (2023), Paper No. 114984, 21 pp.
- [2] R. Diaz and E. Pariguan, On hypergeometric functions and Pochhammer k -symbol. Divulg. Mat. 15 (2007), no. 2, 179–192.
- [3] S. Mubeen and G. M. Habibullah, An integral representation of some k -hypergeometric functions. Int. Math. Forum 7 (2012), no. 1-4, 203–207.



(Poster Presentation)

Approximation by Generalized Abel-Poisson Means of Hexagonal Fourier Series

Aslı Taşkan⁽¹⁾ and Ali Güven⁽²⁾

⁽¹⁾*Balıkesir University, Department of Mathematics, Balıkesir, Türkiye*
e-mail: aslitaskan1016@hotmail.com

⁽²⁾*Balıkesir University, Department of Mathematics, Balıkesir, Türkiye*
e-mail: guvennali@gmail.com

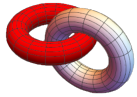
Abstract

This seminar focuses on the approximation of hexagonal Fourier series using generalized Abel-Poisson means. It introduces Hilbert and Banach spaces, orthonormal bases, Fourier coefficients, and Fourier series as the mathematical foundation. It explains the Poisson kernel, Abel-Poisson means, Hölder spaces, generalized Abel means, and the Sharma-Varma theorem. The seminar then extends these ideas to hexagonal Fourier series and reviews approximation results by Ali Güven. The main goal is to obtain a Sharma-Varma type theorem for generalized Abel-Poisson means on hexagonal domains. This study contributes to approximation theory and has potential applications in harmonic analysis, signal processing, image processing, and numerical analysis.

Keywords: Hexagonal domain, Fourier series, Abel-Poisson means.

References:

- [1] P. L. Butzer and R. J. Nessel, Fourier Analysis and Approximation. Pure Appl. Math., Vol. 40, Academic Press, New York-London, 1971.
- [2] A. Guven, Approximation by $(C, 1)$ and Abel-Poisson means of Fourier series on hexagonal domains. Math. Inequal. Appl. 16 (2013), no. 1, 175–191.
- [3] A. Guven, Approximation by means of hexagonal Fourier series in Hölder norms. J. Class. Anal. 1 (2012), no. 1, 43–52.
- [4] Y. Xu, Fourier series and approximation on hexagonal and triangular domains. Constr. Approx. 31 (2010), no. 1, 115–138.
- [5] H. Li, J. Sun and Y. Xu, Discrete Fourier analysis, cubature and interpolation on a hexagon and a triangle. SIAM J. Numer. Anal. 46 (2008), no. 4, 1653–1681.
- [6] S. Sharma and S. K. Varma, Hölder continuous functions and their Abel and logarithmic means. Tamkang J. Math. 30 (1999), no. 3, 167–173.



(Poster Presentation)

Approximation by Deferred Means of Hexagonal Fourier Series

Emine Merve Arslan⁽¹⁾ and Ali Güven⁽²⁾

⁽¹⁾ *Balıkesir University, Department of Mathematics, Balıkesir, Türkiye*
e-mail: mervearslan.ema@gmail.com

⁽²⁾ *Balıkesir University, Department of Mathematics, Balıkesir, Türkiye*
e-mail: guvennali@gmail.com

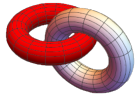
Abstract

In this study, the convergence and approximation properties of Fourier series defined on two-dimensional hexagonal domains are investigated. The rate of approximation results obtained in the literature for classical de la Vallée Poussin means via linear positive operators are generalized in this work to "Deferred de la Vallée Poussin" means, which offer a more flexible structure based on parameter sequences. To overcome the analytical difficulties on the hexagonal lattice, the deferred means of hexagonal Fourier series are constructed by utilizing the intersection geometry and the modulo 3 congruence relation provided by the three-dimensional homogeneous coordinate system. Finally, the rates of approximation (error bounds) of these newly defined operators to continuous functions in the Lipschitz class are proved in terms of the modulus of continuity.

Keywords: Hexagonal domain, Fourier series, rate of approximation, deferred de la Vallée Poussin means.

References:

- [1] X. Z. Krasniqi, Applications of the deferred generalized de la Vallée Poussin means in approximation of continuous functions. *Stud. Univ. Babeş-Bolyai Math.* 67 (2022), no. 4, 829–840.
- [2] P. Chandra, Degree of approximation by generalized de la Vallée-Poussin operators. *Indian J. Math.* 29 (1987), no. 1, 85–88.
- [3] B. Fuglede, Commuting self-adjoint partial differential operators and a group theoretic problem. *J. Functional Analysis* 16 (1974), 101–121.
- [4] A. Guven, Approximation by $(C, 1)$ and Abel-Poisson means of Fourier series on hexagonal domains, *Math. Inequal. Appl.* 16 (2013), no. 1, 175–191.
- [5] A. Guven, Approximation of continuous functions by matrix means of hexagonal Fourier series. *Results Math.* 73 (2018), no. 1, Paper No. 18, 25 pp.
- [6] A. Guven, Approximation of continuous functions by de la Vallée-Poussin means of Fourier series on hexagonal domains. *Jaen J. Approx.* 5 (2013), no. 1, 61–80.
- [7] H. Li, J. Sun and Y. Xu, Discrete Fourier analysis, cubature and interpolation on a hexagon and a triangle. *SIAM J. Numer. Anal.* 46 (2008), no. 4, 1653–1681.
- [8] Y. Xu, Fourier series and approximation on hexagonal and triangular domains. *Constr. Approx.* 31 (2010), no. 1, 115–138.



(Poster Presentation)

Some Approximation Results for Lototsky Type Bernstein-Schurer Operators

Emre Taş⁽¹⁾ and Tuğba Yurdakadim⁽²⁾

⁽¹⁾*Kırşehir Ahi Evran University, Department of Mathematics, Türkiye
e-mail: emretas86@hotmail.com*

⁽²⁾*Bilecik Şeyh Edebali University, Department of Mathematics, Türkiye
e-mail: tugba.yurdakadim@bilecik.edu.tr*

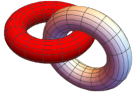
Abstract

In this paper, we study Lototsky type Bernstein-Schurer operators $B_{n,p}^*(f; x)$. These operators generalize the classical Bernstein-Schurer operators by incorporating functions $h_i(x)$ into the Lototsky basis. Using the Korovkin theorem, we prove that the sequence of operators converges uniformly to $f(x)$ on the space of continuous functions. An upper bound for the rate of approximation is obtained using the modulus of continuity. Furthermore, under certain differentiability conditions, a general theorem describing the asymptotic behavior of the operators is proved. These results support and generalize existing studies on Lototsky type operators and positive linear operators in approximation theory.

Keywords: Positive linear operators, Lototsky type Bernstein-Schurer operators, modulus of continuity.

References:

- [1] F. Altomare and M. Campiti, *Korovkin Type Approximation Theory and Its Applications*, De Gruyter, Berlin and New York, 1994.
- [2] R. A. De Vore, *The Approximation of Continuous Functions by Positive Linear Operators*, Springer-Verlag, Berlin, 1972.
- [3] Goldman, Ron, Xiao-Wei Xu and Xiao-Ming Zeng, Applications of the Shorin identity to Bernstein type operators. *Results Math.* 73 (2018), no. 1, Paper No. 2, 26 pp.
- [4] J. P. King, The Lototsky transform and Bernstein polynomials. *Canadian J. Math.* 18 (1966), 89–91.
- [5] S. Kutlu Serin, H. Karsli and F. Taşdelen Yeşildal, Lototsky-Chlodowsky operators. *J. Math. Anal.* 15 (2024), no. 6, 1–11.
- [6] S. Kutlu Serin, H. Karsli and F. Taşdelen Yeşildal, Kantorovich-Stancu type Lototsky-Chlodowsky operators. *Carpathian Math. Publ.* 16 (2024), no. 2, 512–522.
- [7] S. Kutlu Serin, H. Karsli and F. Taşdelen Yeşildal, Durrmeyer type Lototsky-Chlodowsky operators. *Filomat* 39 (2025), no. 13, 4585–4600.
- [8] P. P. Korovkin, *Linear Operators and Approximation Theory*, Hindustan Publishing Corp., Delhi, 1960.
- [9] D. Popa, Intermediate Voronovskaja type results for the Lototsky–Bernstein type operators. *Rev. R. Acad. Cienc. Exactas Fis. Nat. Ser. A Mat. RACSAM* 114 (2020), no. 1, Paper No. 12, 9 pp.
- [10] F. Schurer, *On linear positive operators in approximation theory*, Dissertation, Technological University of Delft, 1965.



(Poster Presentation)

Determination of a Radially Symmetric Potential in an Inverse Scattering Problem

Imene Khélifa⁽¹⁾, Lahcene Chorfi⁽¹⁾ and Ibtissem Djerrar⁽¹⁾

⁽¹⁾ *Badji-Mokhtar Annaba University, Annaba, Algeria*
e-mail: imene.khelifa@univ-annaba.dz

Abstract

This talk concerns an inverse problem in scattering of plane waves by a medium. The forward problem consists to find the far field pattern (or scattering amplitude) from the scatterer (inhomogeneity in the medium). The total wave $u = u^s + u^i$ satisfies the Helmholtz equation

$$\Delta u + k^2 p(x)u = 0 \quad \text{in } \mathbb{R}^2$$

with the radiation condition

$$\lim_{r \rightarrow +\infty} \sqrt{r} \left(\frac{\partial u^s}{\partial r} - iku^s \right) = 0.$$

For incident plane wave $u^i(x) = \exp(ikd \cdot x)$, the far-field $u^\infty(\hat{x})$ is defined by the asymptotic formula

$$u^s(x) = \frac{e^{ikr}}{\sqrt{r}} \left\{ u^\infty(\hat{x}) + \mathcal{O}\left(\frac{1}{r}\right) \right\}, \quad r = |x|, \quad \hat{x} = \frac{x}{r} = (\cos \theta, \sin \theta).$$

More precisely we have ([2])

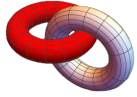
$$u^\infty(\hat{x}) = e^{i\frac{\pi}{4}} \sqrt{\frac{k^3}{8\pi}} \int_{\mathbb{R}^2} e^{ik\hat{x} \cdot y} q(y) u(y) dy \quad (q = 1 - p \in C_0^2(\mathbb{R}^2)).$$

We assume that $p(x) = p(r)$ such that $p(r) = 1$ for $r > R$. The aim of this work is to derive the forward operator $F : q \mapsto u^\infty(\theta) = \sum_{n=0}^{\infty} S_n(q) \cos n\theta$. Then we study the inverse problem (determining the potential q from the far field) which is equivalent to the nonlinear equation $F(q) = u_{ob}^\infty$ where u_{ob}^∞ is the observed far field.

Keywords: Inverse scattering, radial symmetry, far field data.

References:

- [1] J. Shin and E. Arhin, Determining radially symmetric potential from near-field scattering data. J. Appl. Math. Comput. 62 (2020), no. 1-2, 511–524.
- [2] F. Cakoni and D. Colton, A Qualitative Approach to Inverse Scattering Theory. Appl. Math. Sci., Vol. 188, Springer, New York, 2014.



(Poster Presentation)

Parameters Estimation of a Spatio-temporal ARCH Model with Location Dependent Parameters

Karima Kimouche

20 August 1955 University, Department of Mathematics, Skikda, Algeria
e-mail: k.kimouche@univ-skikda.dz

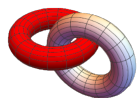
Abstract

In this paper, we define a spatio-temporal ARCH model with location dependent parameters to describe temporal variation and spatial nonstationarity see [1]. We propose two methods for estimating the ARCH functions $a_j(\cdot)$. Both methods are based on a localised least squares criterion see [2]. The first estimator is a localised least squares estimator with constant regressors, whereas the second estimator is a local linear least squares estimator. The sampling properties of these estimators are investigated.

Keywords: Spatio-temporal ARCH, least squares estimator, localised square estimator.

References:

- [1] A. Aouri, M. Boukeloua and S. Kharfouchi, Probabilistic insights into spatio-temporal GARCH model with location-dependent parameters. *TWMS J. App. and Eng. Math.* 15 (2025), no. 6, 1473–1486.
- [2] S. Subba Rao, Statistical analysis of a spatio-temporal model with location-dependent parameters and a test for spatial stationarity. *J. Time Ser. Anal.* 29 (2008), no. 4, 673–694.



(Poster Presentation)

Outlier Detection in the Approximation of Tension Strain Energy of C_{44} Fullerenes: An Approach Based on Fragmental Matrix Property Indices and Non-Parametric Residual Analysis

Oana Moldovan^(1,2) and Lorentz Jäntschi^(1,2)

⁽¹⁾ *Technical University of Cluj-Napoca, Department of Physics and Chemistry, Cluj-Napoca, Romania*
e-mail: Oana.Moldovan@campus.utcluj.ro
e-mail: lorentz.jantschi@chem.utcluj.ro

⁽²⁾ *European University of Technology (eUT+ consortium), European Union*

Abstract

The functional approximation of quantum mechanical properties over discrete isomer spaces frequently encounters outliers that distort predictive stability. This work proposes a mathematical methodology to isolate these anomalies in modeling the Tension Strain Energy (TSE) of the complete set of C_{44} fullerene isomers, utilizing operators based on Fragmental Matrix Property Indices (FMPI) [1].

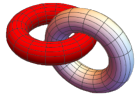
Unlike classical filtering based on standardized residual distances, our approach treats outlier identification as a compliance problem of the error distribution within the approximation manifold. The impact of systematically eliminating specific isomers is critically evaluated through non-parametric tests based on the empirical distribution function of residuals [2]: Anderson-Darling (AD), Kolmogorov-Smirnov (KS), Cramér-von Mises (CM), Kuiper (KV), and Watson U^2 (WU). The underlying error dynamics are monitored computationally via structural tracking indicators (G0, G1, H1, WS, and SF).

Numerical precision and convergence difficulties in standard environments (Python) were resolved by developing a native computing engine in PHP, optimized for fit integrals on finite samples. The concerted use of AD and KV statistics successfully isolated isomers defying the linear approximation. For the TSE property, the most distant outlier exhibits C_1 symmetry, followed by specific structures belonging to the D_2 , C_s , D_3 , S_4 , and T symmetry groups. The methodology provides an objective selection criterion aligned with numerical approximation theory, overcoming the limitations of subjective data trimming in structural modeling [3].

Keywords: Approximation theory, residual analysis, C_{44} fullerenes, non-parametric tests, fragmental matrix property indices (FMPI), PHP.

References:

- [1] L. Jäntschi, Oxidation potential of 2,6-dimethyl-1,4-dihydropyridine derivatives estimated by structure descriptors. *Symmetry* 16 (2024), no. 10, Article No. 1320, 20 pp.
- [2] L. Jäntschi, Detecting extreme values with order statistics in samples from continuous distributions. *Mathematics* 8 (2020), no. 2, Article No. 216, 21 pp.
- [3] S. D. Bolboacă and L. Jäntschi, Modelling the property of compounds from structure: statistical methods for models validation. *Environ. Chem. Lett.* 6 (2008), 175–181 .



(Poster Presentation)

On Fourier transforms of Confluent Hypergeometric Functions and Ramanujan-Type Formulas

Nimete Sh. Berisha⁽¹⁾, Faton M. Berisha⁽²⁾ and Bujar Xh. Fejzullahu⁽³⁾

⁽¹⁾ *University of Prishtina, Faculty of Economics, Prishtina, Kosovo*
e-mail: nimete.berisha@uni-pr.edu

⁽²⁾ *University of Prishtina, Department of Mathematics, Prishtina, Kosovo*
e-mail: faton.berisha@uni-pr.edu

⁽³⁾ *University of Prishtina, Department of Mathematics, Prishtina, Kosovo*
e-mail: bujar.fejzullahu@uni-pr.edu

Abstract

We derive some Fourier transforms of confluent hypergeometric functions. We give generalizations of several well-known results involving Fourier transforms of gamma functions. In particular, the generalizations include several of Ramanujan's remarkable formulas.

Our first result is the following theorem:

Theorem. For $a, \alpha, z \in \mathbb{C}$, with $\Re(b) > 0$ and $\Re(c) > 0$,

$$\begin{aligned} & \int_{-\infty}^{+\infty} \frac{\Gamma(b+it)}{\Gamma(b+c+it)} {}_2F_2(a, c; \alpha, b+c+it; z) e^{it\xi} dt \\ &= \begin{cases} \frac{2\pi}{\Gamma(c)} e^{-b\xi} (1-e^{-\xi})^{c-1} {}_1F_1(a; \alpha; z(1-e^{-\xi})), & \xi > 0, \\ 0, & \xi < 0. \end{cases} \end{aligned}$$

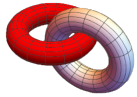
In particular, for $\alpha = c$

$$\begin{aligned} & \int_{-\infty}^{+\infty} \frac{\Gamma(b+it)}{\Gamma(b+c+it)} {}_1F_1(a; b+c+it; z) e^{it\xi} dt \\ &= \begin{cases} \frac{2\pi}{\Gamma(c)} e^{-b\xi} (1-e^{-\xi})^{c-1} {}_1F_1(a; c; z(1-e^{-\xi})), & \xi > 0, \\ 0, & \xi < 0. \end{cases} \end{aligned}$$

Keywords: Confluent hypergeometric function, Fourier transform, Ramanujan integrals.

References:

- [1] N. Sh. Berisha, F. M. Berisha and B. Xh. Fejzullahu, Some Fourier transforms involving confluent hypergeometric functions. *Integral Transforms Spec. Funct.* 35 (2024), no. 5-6, 316-329.
- [2] B. C. Berndt, *Ramanujan's Notebooks, Part II*. Springer, New York, 2011.
- [3] F. W. J. Olver, D. W. Lozier, R. F. Boisvert and C. W. Clark, *NIST Handbook of Mathematical Functions*. Cambridge Univ. Press, Cambridge, 2010.



(Poster Presentation)

Numerical Approach for the Analysis and Optimization of GI/M/1/N Queue with Vacations

Sofiane Ouazine⁽¹⁾ and Karim Abbas⁽²⁾

⁽¹⁾ *University of Bejaia, Faculty of Exact Sciences, LMA laboratory, Bejaia, Algeria*
e-mail: sofiane.ouazine@univ-bejaia.dz

⁽²⁾ *Bejaia University, Department of Operation Research, Unit LAMO, Bejaia, Algeria*
e-mail: karim.abbas@univ-bejaia.dz

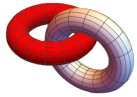
Abstract

The main purpose of the current paper is to use Taylor series expansion approach for analyzing and optimization of the GI/M/1/N queue with multiple working vacations, with an emphasis on perturbation analysis. We then considered a non-parametric sensitivity analysis to investigate the numerical evaluation of stationary characteristics of the same queue in the case where we suppose that the working vacation rate is not assessed in perfect manner, i.e. it is subject to propagate uncertainty. Furthermore, an approximative values of the expectation and variance of some stationary characteristics have been also obtained. This approach has the advantage of being transform-free as it avoids the use of Laplace transforms and/or numerical inversion techniques, which are predominantly used in the literature. Several numerical examples are also presented for illustrative purposes.

Keywords: Multiple working vacations, Taylor series expansion, numerical approximation, optimization, parametric uncertainty.

References:

- [1] K. Abbas, B. Heidergott and D. Aissani, A functional approximation for the M/G/1/N queue. *Discrete Event Dyn. Syst.* 23 (2013), no. 1, 93–104.
- [2] Y. Baba, Analysis of a GI/M/1 queue with multiple working vacations. *Oper. Res. Lett.* 33 (2005), no. 2, 201–209
- [3] A. D. Banik, U. C. Gupta and S. S. Pathak, On the GI/M/1/N queue with multiple working vacations- analytic analysis and computation. *Appl. Math. Modell.* 31 (2007), no. 9, 1701–1710.
- [4] X.-R. Cao, The Maclaurin series for performance functions of Markov chains. *Adv. in Appl. Probab.* 30 (1998), no. 3, 676–692.
- [5] B. Heidergott and A. Hordijk, Taylor expansions for stationary Markov chains. *Adv. in Appl. Probab.* 35 (2003), no. 4, 1046–1070.



(Poster Presentation)

Approximation Results for Chlodowsky Modification of A New Bernstein-like Operators

Tuğba Yurdakadim⁽¹⁾ and Emre Taş⁽²⁾

⁽¹⁾ *Bilecik Şeyh Edebali University, Department of Mathematics, Türkiye*
e-mail: tugba.yurdakadim@bilecik.edu.tr

⁽²⁾ *Kırşehir Ahi Evran University, Department of Mathematics, Türkiye*
e-mail: emretas86@hotmail.com

Abstract

In the present paper, we introduce the Chlodowsky modification of a new Bernstein-like operators which is recently defined in [6]. We examine the approximation properties dealing with the rate of convergence in terms of modulus of continuity and Peetre-K functional. Under the conditions of differentiability of a given function f or belonging to Lipschitz class, we obtain upper bounds for $\|L_{r,a,b}(f) - f\|_{C[-a,(1+a)b_r-a]}$.

Keywords: Positive linear operators, Bernstein-Chlodowsky operators, modulus of continuity, Lipschitz space, Peetre K-functional.

References:

- [1] F. Altomare and M. Campiti, Korovkin Type Approximation Theory and Its Applications, De Gruyter, Berlin and New York, 1994.
- [2] I. Chlodowsky, Sur le developpement des fonctions definies dans un intervalle infini en series de polynomes de M. S. Bernstein. *Compositio Math.* 4 (1937), 380–393.
- [3] N. L. Braha, V. Loku, T. Mansour and M. Mursaleen, A new weighted statistical convergence and some associated approximation theorems. *Math. Methods Appl. Sci.* 45 (2022), no. 10, 5682–5698.
- [4] Z. Ditzian and V. Totik, Moduli of Smoothness, vol 9 of Springer Series in Computational Mathematics, Springer-Verlag, New York, 1987.
- [5] A. İzgi, Approximation by a class of new type Bernstein polynomials of one and two variables. *Global J. Pure Appl. Math.* 8 (2012), no. 5, 55–71.
- [6] A. İzgi and D. Dinsever Karahan, On the Korovkin-type and statistical approximation of a new linear positive operators. *Commun. Fac. Sci. Univ. Ank. Ser. A1. Math. Stat.* 75 (2026), no. 1, 14–25.
- [7] H. Karsh and V. Gupta, Some approximation properties of q -Chlodowsky operators. *Appl. Math. Comput.* 195 (2008), no. 1, 220–229.